Informational Efficiency and Liquidity Premium as the Determinants of Capital Structure

Chun Chang and Xiaoyun Yu*

Abstract

This paper investigates how a firm’s capital structure choice affects the informational efficiency of its security prices in the secondary markets. We identify two new determinants of a firm’s capital structure policy: the liquidity (adverse selection) premium due to investors’ anticipated losses to informed trading, and operating efficiency improvement due to information revelation from the firm’s security prices. We show that the capital structure decision affects traders’ incentives to acquire information and subsequently, the distribution of informed traders across debt and equity claims. When information is less imperative for improving its operating decisions, a firm issues zero or negative debt (i.e., holding excess cash reserves) in order to reduce socially wasteful information acquisition and the liquidity premium associated with it. When information is crucial for a firm’s operating decisions, the optimal debt level is one that achieves maximum information revelation at the lowest possible liquidity cost. Our model can explain why many firms consistently hold no debt. It also provides new implications for financial system design and for the relationship among leverage, liquidity premium, profitability, and the cost of information acquisition.

I. Introduction

At least since Hayek (1945), economists have been aware of a positive allocative role of information conveyed by prices. In recent years, an increasing number of researchers have recognized and emphasized the effect of security prices on production and investment decisions in the economy: Security prices

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can credibly convey certain types of information that help firms to make efficient operating decisions (e.g., Hirshleifer (1971), Allen (1993), Holmström and Tirole (1993), Dow and Gorton (1997), Subrahmanyam and Titman (1999), and Chang and Yu (2004)). In this paper, our primary objective is to investigate whether and how an individual firm’s capital structure can be designed to increase this informational (operating) efficiency.

The cost of information production must be borne by someone. This cost takes the form of liquidity premiums in security returns when security buyers require compensation in anticipation of having to trade with informed traders later. Given the central importance of a firm’s cost of capital in determining its value, many suggestions have been made on how to reduce liquidity premiums (e.g., Amihud and Mendelson (2000)). The second objective of our paper is to examine the impact of a firm’s capital structure policy on the liquidity premiums of its securities.\(^1\)

We explore to what extent a firm’s capital structure policy affects the operating efficiency and liquidity premium in the context of a (market microstructure) trading framework. Unlike most models, however, informed traders in our setting optimally choose to acquire information and to allocate trades between a firm’s shares and bonds, depending on both the informational sensitivity and liquidity trading associated with these two securities. In addition, the firm’s optimal operating decision is contingent on the information conveyed by the two security prices.\(^2\)

The underlying intuition of our analysis is as follows: A firm’s capital structure decision affects the information sensitivity and the liquidity trading of the securities it issues and, consequently, the distribution of informed traders across debt and equity claims. Increasing leverage to a certain level can make a firm’s equity more informationally sensitive and thus informed trading more profitable for the wealth-constrained traders. This in turn increases the incentive to acquire information. Further, traders’ information acquisition efforts have two opposite effects on the firm’s value. On the one hand, informed trading is costly to the firm, as uninformed investors demand extra compensation in the form of a liquidity premium when they purchase the firm’s securities in anticipation of their losses to the informed traders. On the other hand, information revelation in the secondary markets may help to improve the firm’s operating decisions. The optimal capital structure rests on the trade-off between improving operating efficiency and reducing liquidity premium.

We show that when information is less imperative for its operating decisions, a firm prefers to issue zero or even a negative amount of debt (i.e., holding excess cash reserves). In this case, the benefit of information revelation in the capital

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\(^1\)It should be noted that the concept of liquidity has been used more broadly in the literature. The liquidity premiums in our model are only due to the effect of adverse selection in the secondary markets. We do not address liquidity premiums from other perspectives in this paper.

\(^2\)Our paper uses a variation of the Kyle (1985) trading mechanism. Beyond the issue that informed traders optimally choose to allocate their trades between the two securities, the normality condition in the standard Kyle model is violated when we split the firm’s payoff into a nonlinear debt and a nonlinear equity payoff. These difficulties prevent us from adopting a general security design approach. We restrict the firm to issuing only debt and equity. Since most firms’ capital structures consist of only debt and equity, this is a reasonable starting point.
markets is economically negligible because informed trading does not generate a significant gain in the firm’s operating efficiency. The optimal capital structure reduces socially wasteful information acquisition and its associated liquidity premium. By using no debt or even adding cash to the equity holders’ payoff (negative debt), the equity becomes so “safe” that traders have little incentive to acquire information.

If, instead, information revelation in the capital markets can substantially improve its operating decisions, a firm issues debt to achieve the maximum level of information revelation at the lowest possible liquidity cost. In this case, the benefit due to an improved operating decision is maximized and the loss sustained by informed investors is minimized at the same level of debt—a point where a marginal informed trader is indifferent between trading bonds and trading equity. At this debt level, the debt is no longer default risk-free; however, it is not risky enough to induce any informed trading in the bond market. With all the informed trading still concentrated in the stock market, intense competition among informed traders minimizes their trading profits, and maximum information revelation occurs. Had leverage been raised beyond this point, debt would have become risky enough that some informed traders would have gravitated toward the debt market. This leads to a lower value for the firm, because it engenders a liquidity premium in the firm’s debt securities and makes the liquidity premium in its stock higher due to less competition among informed traders. In addition, it makes the prices of all of the firm’s securities less informative.

This paper contributes to the literature on the “underleverage” puzzle, which argues that many firms borrow less than what the major theories of capital structure predict (e.g., Miller (1977), Myers (2003)). While the dominant view in the capital structure literature is that financial distress cost drives the cost of debt financing, the significance of financial distress cost itself has been questioned (Haugen and Senbet (1988)). Even if this cost is significant, firms appear to be underleveraged in the presence of the tax benefits of debt financing (Graham (2000)). Even more puzzling, a significant fraction of firms persistently hold no debt or hold excess cash reserves (Opler, Pinkowitz, Stulz, and Williamson (1999), Strebulaev and Yang (2006)).

Our model sheds light on these puzzling findings by establishing that in the presence of liquidity costs and informational benefits, a firm’s optimal debt level can hover around two levels: a negative debt level aimed at reducing socially wasteful informed trading when it does not contribute to a substantial operating efficiency gain, or a positive debt level aimed at maximizing information revelation when informed trading can improve operating efficiency. Our theory predicts that information revelation is less likely to benefit investment or liquidation decisions for firms with better growth opportunities, more profits, or fewer tangible assets. Therefore, these firms are more likely to hold negative debt. This is consistent with the empirical evidence that firms that are fast-growing, small, and with high research and development (R&D) expenditures tend to hold substantially more cash (Opler et al. (1999), Mikkelson and Partch (2003)). Our model also suggests caution in interpreting the existing empirical results, as these two clusters of firms may have to be separated in empirical analysis (Strebulaev and Yang (2006)).
We also show that the optimal debt level increases in the cost of information acquisition and decreases in information precision. Our results thus imply that, ceteris paribus, firms with a lower cost of information acquisition borrow less and that firms rely less upon debt financing if their information environments improve over time. While we are not aware of any empirical work directly testing these implications, which are particular to our model, they appear to be testable. Additionally, our model provides empirical implications for a firm’s liquidity premium, the number of informed traders, and the information efficiency of its security prices.

The rest of the paper is organized as follows. In Section II we discuss the studies that form the background to our paper. In Section III we lay out the model. Section IV analyzes informed trading behavior for a given level of debt. Section V examines the determinants of a firm’s capital structure. Section VI discusses the empirical implications of the paper. Section VII provides several extensions. Section VIII concludes. All definitions of symbols, proofs, and figures are given in Appendix A.

II. Related Literature

Our paper is related to the recent theoretical research analyzing the effects of security prices on firms’ real decisions, such as designing efficient corporate compensation contracts (Holmström and Tirole (1993)), ownership structure at the time of the initial public offering (IPO) (Maug (2001)), public financing decisions (Subrahmanyam and Titman (1999)), and evaluating the efficiency of financial systems among different countries (Boot and Thakor (1997), Rajan and Zingales (1998)). Instead, we investigate the effect of informational efficiency in the context of a firm’s capital structure decision. The basic premise of our model also finds support in the empirical evidence that firms’ operating decisions are affected by the informational efficiency of their security prices (e.g., Morck, Yeung, and Yu (2000), Wurgler (2000), Durnev, Morck, and Yeung (2004), Giammarino, Heinkel, Hollifield, and Li (2004), Bakke and Whited (2006), and Chen, Goldstein, and Jiang (2007)) and that the going-public process generates a considerable amount of firm-specific information that is absent from the issuer’s information set (e.g., Hanley (1993), Cornelli and Goldreich (2003)).

Substantial theoretical work and empirical evidence have now established that the liquidity premium in a firm’s security returns is a significant and sizable component of its cost of capital. Additionally, the adverse selection problem in the secondary markets due to the presence of privately informed traders is a primary cause of illiquidity (e.g., Brennan and Subrahmanyam (1996), Bhattacharya and Daouk (2000), Easley, Hvidkjaer, and O’Hara (2002), and Easley and O’Hara (2004)). Liquidity is also believed to be a primary impetus for the waves of financial innovation in recent decades (Gorton and Pennacchi (1990), Subrahmanyam (1991), and Boot and Thakor (1993)). We contribute to this literature by analyzing how a firm’s capital structure policy affects the liquidity premium associated with its securities. In this respect, our results are consistent with Lesmond, O’Connor, and Senbet (2008), who document that increases (decreases) in debt usage accomplished using leverage recapitalizations are significantly associated with increases (decreases) in liquidity cost and in information asymmetry in the remaining equity.
Our paper also builds on the literature analyzing how splitting cash flows affects information acquisition and liquidity (e.g., Gorton and Pennacchi (1990), Boot and Thakor (1993)) and how securities can be designed to induce information revelation (e.g., Fulghieri and Lukin (2001), Habib and Johnsen (2000)). Differing from the former, however, we consider the effect of informed trading upon a firm’s operating efficiency. We show that reducing debt (even to a negative level), rather than issuing more default risk-free debt, can make uninformed investors more willing to buy equity claims. Differing from the latter, we concentrate on information revelation in the secondary markets instead of information asymmetry and information acquisition at the time when securities are issued. Our contribution is therefore significant for three reasons. First, our research focus is important because information revelation occurs more frequently in the secondary markets for most listed firms whose securities are continuously traded, whereas security issuances to the capital markets are sporadic. Even if there is a decision to issue new securities, most information will be revealed in the secondary markets prior to the actual issuance of the securities. Second, focusing on the secondary markets allows us to explore the effect of capital structure policy on liquidity premiums associated with debt and equity. Our paper hence establishes a formal link between firms’ financial decisions and their security returns that is usually absent in the literature. Lastly, we concentrate on what target capital structure a firm should maintain in order to achieve the informational efficiency of its security prices, rather than on what optimal security it should issue to convey the information it possesses at the time of the offer. Since information and liquidity considerations enter into the design of a firm’s target capital structure, the costs and benefits associated with information revelation are not transitory in nature.3

III. The Model

There are three dates. At date 0, the firm chooses its capital structure (debt payment level) $F$ to maximize the value that investors are willing to pay. At date 1, some of the date 0 investors suffer liquidity shocks, and trades in the secondary markets take place. The firm’s operating decision is made immediately after the prices in the secondary markets are determined. At date 2, the securities’ payoffs are realized.

A. The Firm

The value of the firm at date 2 depends on its operating decision made at date 1. To simplify the exposition, we interpret a firm’s operating decision in the context of whether or not to liquidate (restructure). The value of the firm is $m + \varepsilon$

3It should be emphasized that although our paper identifies two potentially significant factors in determining a firm’s capital structure, we do not claim that other factors are unimportant. Due to space limitations, we can only discuss the papers that are closely related to our approach. For a comprehensive review of the vast literature of corporate capital structure decisions, see Harris and Raviv (1991) and Myers (2003).
if there is no liquidation, and \( L > 0 \) if liquidation occurs. Now \( \varepsilon \) is either \( e \) or \(-e\) with equal probability of \( \frac{1}{2} \). To avoid unlimited liabilities on equityholders, we assume \( m \geq e \). To focus on the case where information is useful for a firm’s operating decision, we assume \( m - e \leq L < m \). That is, liquidation is not always preferred over continuation; it is (weakly) preferred only when the bad state (\( \varepsilon = -e \)) occurs. In particular, when \( m - e = L \), information revelation does not generate any value for the firm.\(^4\)

At date 1, the firm’s operating decision is based on the information conveyed in its bond and stock prices. There are three justifications for why the firm may need this information to make an efficient decision. First, the managers may not have all the information about the future conditions of the firm’s input and output markets.\(^5\) Second, incompetent managers may be either unaware of their own incompetence or aware but unwilling to abdicate. The board of directors may use this information to make changes in management. Lastly, the managers may be well informed, but due to the agency problem suggested by Jensen (1986), they may prefer taking on a growth opportunity regardless of its NPV. Again, the board can use this information (that the stock price is too low, for example) as justification to block new investment.\(^6\)

B. Investors at Date 0

At date 0, there is a continuum of investors who are willing to buy the firm’s securities. A random fraction, \( \theta \), of date 0 investors will experience liquidity shocks at date 1, so they will have to consume immediately by selling the securities they hold. A \( 1 - \theta \) fraction of date 0 investors will consume at date 2. For tractability, we assume that the random variable \( \theta \) is uniformly distributed on \([\bar{\theta} - z, \bar{\theta} + z]\) with \( \bar{\theta} + z \leq 1 \), where \( z \) measures the dispersion of the liquidity shock.

To what extent should liquidity trading in each of the two markets change as the leverage level changes? Previous market microstructure models with multiple markets are unable to completely address the issue because they often treat liquidity trading in each market exogenously as a random noise variable. Instead, we follow Diamond and Dybvig (1983) to let \( \theta \) measure the investors’ aggregate liquidity risk and model liquidity trading as proportional to the amount of security issued. Since \( \theta \) fraction of date 0 investors experience liquidity shocks at date 1

\(^4\)Our setup is not limited to liquidation decisions. Two alternative interpretations of operating decisions are equally applicable. One can interpret \( m + e \) as the firm’s value under the current management and \( L \) as the value under a new management team. Another interpretation is that it is the firm’s decision whether or not to take on its growth opportunities. In this case, \( m + e - L \) is the net present value (NPV) of its growth (or merger and acquisition) opportunities, and \( L \) is the value of its assets in place. The assumption of \( m - e \leq L \) can then be interpreted as the NPV of the growth opportunity in the case of the bad state (\( m - e - L \)) being nonpositive.

\(^5\)Many in the literature have used this assumption (e.g., Allen (1993), Boot and Thakor (1997), Dow and Gorton (1997), and Subrahmanyam and Titman (1999)). This assumption also finds support in the empirical evidence on the IPO process.

\(^6\)For this to be the case, the managerial compensation scheme has to be ineffective. One way in which it becomes ineffective is when managers are wealth-constrained. In order to induce efficient investment, managers have to be given a large stake in their firms. This may be too costly to investors.
and have to sell their securities at the market price, the dollar amount of liquidity trading in the stock (bond) market is $\theta S_1 \left( \theta B_1 \right)$, where $S_1 \left( B_1 \right)$ is the date 1 market value of the stock (bond).\footnote{When investors who suffer liquidity shock sell their holdings in the security markets, the dollar value of the order flow that the market makers receive is the number of shares multiplied by the current market price. Ideally, one should model the price formation process (e.g., how $S_1$ is determined when the market makers observe the number of shares submitted). However, under the concept of rational expectations equilibrium and the pricing mechanism in which market makers set the price, using the number of shares is equivalent to using the dollar value as a measure of order flow in equilibrium because one can be inferred from the other when the price is set.} Despite the fact that the distribution of $\theta$ is exogenously given, this formulation allows the dollar amount of liquidity trading in each market to depend continuously on the market value of the security and, therefore, to be endogenous to the choice of capital structure. It is also consistent with the empirical evidence suggesting that changes in capital structure alter the degree of liquidity trading (Lesmond et al. (2008)) and that firms whose stocks are most liquid have lower leverage than those whose stocks are least liquid (Lipson and Mortal (2009)).\footnote{Gorton and Pennacchi (1993) allow investors to have heterogeneous liquidity preferences and endowment distributions. For trackability, we abstract from that dimension and instead focus on how informed traders can choose to trade optimally between the stock and bond markets. Nevertheless, our model can be generalized to allow for the heterogeneity in liquidity trading by setting different distributions of $\theta$ among investors. But this is analogous to assuming two $\theta$ distributions across the two security markets. The market makers’ Bayesian inference will then be based on two $\theta$ distributions. Our main results, however, will still carry through even though a closed-form solution is not available.}

It will be clear later that our specification allows for an explicit characterization of the liquidity premium required by date 0 investors to compensate for their potential loss to the informed traders. Unlike the “noise traders” formulation in a typical market microstructure setup, we fully account for all agents’ gains or losses. Consequently, investors are indifferent between purchasing debt and equity at date 0.

C. Investors at Date 1

Each investor at date 1 has an endowment of one dollar. A date 1 investor can acquire information about $\varepsilon$ at cost $c$ and receive a noisy signal that reveals $\varepsilon$ correctly with probability $q > \frac{1}{2}$. If a date 1 investor does not spend $c$, he remains uninformed. Uninformed date 1 investors can become market makers, provided the rate of return they earn is not less than 0. In equilibrium, they are willing to buy or sell shares when the expected payoff equals investment.

Similar to Boot and Thakor (1993), we assume that an informed investor faces borrowing constraints and can trade securities only up to his endowment. This assumption is sensible because the borrowers are better informed.\footnote{This endowment constraint prevents any (risk-neutral) informed investor from trading an unlimited amount through borrowing. In reality, such a constraint does exist. Note also that the need for liquidity rules out the standard argument that homemade leverage can undo the firm’s leverage. To apply that argument, we would have to allow date 0 investors to issue homemade publicly traded debt. The cost of doing so is prohibitively high because one would have to assess all date 0 investors’ credit histories.} One can motivate this assumption in many ways following the credit rationing literature.
In equilibrium, information acquisition is optimal for date 1 investors only if the expected profit from informed trading is sufficiently large to compensate for the cost of information acquisition.\(^{10}\) The equilibrium number of informed traders, denoted as \(\phi\), is then endogenously determined by equating the expected informed trading profit with the cost of information acquisition, \(c\).

**D. Pricing Mechanism and Definition of Equilibrium**

At date 1, market makers in each market observe the total order flow submitted by both liquidity investors and informed investors, and they set the competitive price to break even. In this respect, the security prices are determined similarly to an auction market, as in Kyle (1985). Different from Kyle (1985), however, our model allows the firm’s operating decision, and consequently its value, to be contingent on the information conveyed in its security prices, and it allows the firm’s action to be rationally anticipated by the market makers.\(^{11}\)

In our one-period (static) trading model, market makers in each market set the price simultaneously without observing the price in the other market. Since there has been no general consensus as to how the market makers in the two markets learn from each other, this simplifying assumption is a natural starting point. More importantly, the prices of the two securities are dependent upon each other despite the fact that market makers set them simultaneously. This is because the firm observes the prices from the two markets and conditions its operating decision on the information revealed from these prices; market makers in each market rationally anticipate this when they quote the security price. In Section VII.B, we discuss some implications when this assumption is relaxed.

**Definition 1.** An equilibrium consists of the firm’s capital structure choice, informed traders’ trading strategies, the number of informed traders trading in the bond and equity markets, the market makers’ pricing strategies and their beliefs given order flows, and the firm’s operating decision and beliefs given market makers’ prices such that:

1. the firm chooses the capital structure \(F\) to maximize its value at date 0;
2. the fraction of informed traders in each market is determined so that no informed trader has incentive to trade in the other market;
3. the market makers quote stock and bond prices so that, based on their beliefs about informed traders’ trading strategy and the firm’s optimal operating decision, they break even;
4. the firm makes the optimal operating decision at date 1 based on the information conveyed by the security prices;

\(^{10}\)Our results can be extended to the situation where, at the cost of \(c\), date 0 investors who do not suffer liquidity shocks can become informed at date 1. The “free-entry” assumption of becoming informed and the competition among informed traders ensure that a marginal date 0 investor is indifferent between being informed and remaining uninformed, as his expected payoff would be the same in both cases. Therefore, we can treat the date 0 investors as if they have no choice of whether to be informed or not.

\(^{11}\)Our setting also differs from that of Kyle (1985) in that we have many competitive, informed traders. In Kyle (1985), there is a single informed trader who acts strategically.
v) the beliefs are consistent (satisfying Bays’ rule wherever applicable); and
vi) the expected trading profit of a marginal informed trader equals his cost of information acquisition \( c \).

The sequence of events in the model is summarized in Figure 1. Throughout the paper, we use a superscripted \( S \) and \( B \) to denote stocks and bonds, respectively.

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**FIGURE 1**
Sequence of Events

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Date 1</th>
<th>Date 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm chooses the amount of debt, ( F ). The rest is financed by equity. Both debt and equity are sold to investors.</td>
<td>A random fraction ( \theta ) of date 0 investors suffer liquidity shocks and submit their orders to the market makers.</td>
<td>Payoffs are realized and distributed to investors.</td>
</tr>
<tr>
<td>Informed investors choose in which security market to trade. Market makers observe the total order flow and set security prices as in Kyle (1985)</td>
<td>Firm makes liquidation decision.</td>
<td></td>
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**IV. Informed Trading for a Given Capital Structure \( F \)**

We solve our model with backward induction and derive the equilibrium as follows:

i) For a given debt level \( F \) and the number of informed traders \( \phi \), we postulate that \( \lambda^S \) fraction of the informed traders will trade the stock and \( \lambda^B \) fraction will trade the debt. We then derive the prices set by the market makers and a marginal informed investor’s expected return of trading the stock, \( R^S \), or the bond, \( R^B \).

ii) By comparing \( R^S \) and \( R^B \), we obtain the equilibrium allocation of informed traders, \( \lambda^S \) and \( \lambda^B \). By equating the informed trading profit to the cost of information acquisition, we derive the equilibrium number of informed traders, \( \phi \), and the aggregate profit from trading stock and bonds, \( \pi (F) \).

iii) We then compute the firm’s operating gain due to information revelation in the security markets, \( G(F) \). Since informed trading profits represent the losses to the liquidity traders, the price of each of the firm’s securities that an average date 0 investor is willing to pay is then his expected payoff minus his expected loss to the informed traders in that market. The total value of the firm is \( m + G(F) - \pi (F) \). The optimal capital structure is \( F \) that maximizes the value of the firm.

To focus on the more interesting case, we impose the following assumption:

**Assumption 1.** Assume \( c < \left( (2q - 1) e \right)/m \).
As will become clear later, this condition ensures that the information acquisition cost $c$ is not so high that traders no longer have an incentive to become informed.

Let $S_1$ and $B_1$ denote the date 1 stock price and bond price, respectively, and $\tau$ denote $\phi \frac{(2q - 1)}{z}$. Market makers in each market set competitive price $(S_1, B_1)$ after observing the aggregate order submitted as in Kyle (1985); they cannot distinguish whether the orders are submitted by the informed traders or by the liquidity traders. However, the firm makes inferences from its security prices to help with its operating decision, and its actions are rationally anticipated by the market makers when they quote prices.

Figure 2 illustrates the distributions of order flows in the stock and bond markets. Appendix B presents the derivations of equilibrium security prices. Based on these prices, Lemma 1 describes the informed trading returns in the stock market ($R^S$) and the bond market ($R^B$).

**FIGURE 2**

Overlapping Intervals of Order Flows When Stock and Bond Markets Are Open

If $\varepsilon = \varepsilon$, then order flow is $\theta S_1 - \phi \frac{(2q - 1)}{z} \lambda^S$ in the stock market and is $\theta B_1 - \phi \frac{(2q - 1)}{z} \lambda^B$ in the bond market. If $\varepsilon = -\varepsilon$, then order flow is $\theta B_1 + \phi \frac{(2q - 1)}{z} \lambda^S$ in the stock market and is $\theta B_1 + \phi \frac{(2q - 1)}{z} \lambda^B$ in the bond market. $\theta$ is uniformly distributed at $[\theta - z, \theta + z]$.

**Lemma 1.** Given the debt level $F$, the number of informed traders $\phi$, and the distribution of informed traders between the two markets $(\lambda^B, \lambda^S)$, an informed stock trader’s expected return is

$$R^S = (2q - 1) \left( 1 - \frac{2\tau \lambda^S}{m - F + \max(m, F, e)} \right) \left( \frac{e}{\max(m, F, e)} \right).$$

An informed bond trader’s expected return is

$$R^B = (2q - 1) \left( \frac{F - \min(F, m - e)}{F + \min(F, m - e)} \right) - \frac{2\tau (2q - 1)}{m + e - F} \times \left( \frac{F - \min(F, m - e)}{F + \min(F, m - e)} \right) \lambda^S \lambda^B$$

$$- \frac{2\tau (2q - 1)}{F + m - e} \left( \frac{F - \min(F, L)}{F + \min(F, L)} \right) \lambda^B.$$
That is, if $F \leq m - e$, then
\[ R^S = (2q - 1) \left( 1 - \frac{\tau \lambda^S}{m - F} \right) \left( \frac{e}{m - F} \right) \quad \text{and} \quad R^B = 0. \]

If $F > m - e$, then
\[ R^S = (2q - 1) \left( 1 - \frac{2\tau \lambda^S}{m + e - F} \right). \]

If $m - e < F < L$, then
\[ R^B = (2q - 1) \left( 1 - \frac{2\tau \lambda^S}{m + e - F} \right) \left( \frac{F - (m - e)}{F + m - e} \right). \]

If $F > L$, then
\[ R^B = (2q - 1) \left( \left( 1 - \frac{2\tau \lambda^S}{m + e - F} \right) \left( \frac{F - (m - e)}{F + m - e} \right) \right. \]
\[ + \left( \frac{2\tau \lambda^S}{m + e - F} - \frac{2\tau \lambda^B}{F + m - e} \right) \left( \frac{F - L}{F + L} \right). \]

The first term of equation (1) captures the aggregate precision of the signals. The second term of equation (1) is the probability that the stock price is not fully revealing (the overlapping region in Figure 2). The third term is the sensitivity of equity payoff relative to the information. Lemma 1 indicates that these factors affect the return to informed stock trading, $R^S$.

Lemma 1 also suggests that informed stock trading return does not depend on liquidation value. This is because when the stock price fully reveals the true state, informed traders make no profit; when the stock price is not revealing, the two states of $\varepsilon$ are equally likely and the firm will not be liquidated, as the average value $m$ is greater than $L$.

In contrast, equation (2) indicates that informed bond trading return is contingent on $L$. Since the stock price is more informative than the bond price, in some instances the stock price reveals the true state, but the bond price does not.\textsuperscript{12} Although informed bond trading can still be profitable because the bond price is not fully revealing, this profit depends on whether or not the firm liquidates, and on the liquidation value $L$. Liquidation limits the gain per share traded for a trader who sells bonds in the bad state to $B_1 - L$, smaller than the gain of $B_1 - (m - e)$ per share traded had the liquidation not occurred. Consequently, the informed bond trading return decreases as the fraction of informed stock traders increases (higher $\lambda^S$) because more informed trading in the stock market makes stock price more informative and liquidation in the bad state more likely. Informed bond trading return also decreases in the fraction of informed bond traders (higher $\lambda^B$) as a result of more competition among informed traders in the bond market.

\textsuperscript{12}See the proof of Corollary 1 in Appendix C.
The last part of Lemma 1 suggests that when the bond is default risk-free \((F \leq m - e)\), informed traders never make a profit from trading the bond. Information is conveyed by the stock price while the bond price remains uninformative.

We now derive the equilibrium allocation of informed traders between the two markets by comparing \(R^S\) and \(R^B\) given by Lemma 1. Proposition 1 summarizes the equilibrium allocation \((\lambda^B, \lambda^S)\) as a function of debt level \(F\).

**Proposition 1.** There exist \(\lambda^* (F)\) and \(F^*\) such that, in equilibrium:

i) If \(F \leq F^*\), then \(\lambda^S = 1, \lambda^B = 0\).

ii) If \(F^* < F < m + e\), then \(\lambda^S = \lambda^*, \lambda^B = 1 - \lambda^*,\) and \(\lambda^*(F)\) decreases in \(F\).

iii) If \(F = m + e\), then \(\lambda^S = 0, \lambda^B = 1\).

Specifically,

\[
F^* = \frac{-\alpha + \sqrt{\alpha^2 - 4\beta}}{2} > L,
\]

where

\[
\alpha \equiv \frac{2q - 1 - 3c}{2q - 1 - c} (m - e) - L \quad \text{and} \quad \beta \equiv -L (m - e) \left( \frac{2q - 1 + c}{2q - 1 - c} \right).
\]

Proposition 1 suggests that when the debt level is low, all the informed trading occurs in the equity market and no informed traders trade bonds \((\lambda^S = 1\) and \(\lambda^B = 0)\). In particular, the bond price remains uninformative \((\lambda^B = 0)\) even if the bond is no longer risk-free \((L < F \leq F^*)\). Intuitively, this is because up to the debt level \(F^*\), the bond market has not accumulated sufficient liquidity and the bond is still not informationally sensitive enough. Even with all the informed trading occurring in the stock market, any deviation to bond trading yields a smaller return compared to stock trading, which makes such a deviation not worthwhile. A marginal informed trader would still prefer stock.

When the debt level exceeds \(F^*\), the bond becomes risky enough and sufficient liquidity is generated in the bond market. As a result, \(R^B\) rises to approach \(R^S\). Now it pays for some informed traders to switch to the bond market, making both security prices partially informative. As debt increases further, even more liquidity can be generated in the bond market and more informed traders will be lured away from trading stock. The fraction of informed traders who trade the bond, \(\lambda^B\), is determined such that a marginal informed trader is indifferent between trading bonds and trading stocks \((R^B (\lambda^B) = R^S (\lambda^S)\) and \(\lambda^B = 1 - \lambda^S)\).

Once \(F\) reaches \(m + e\), the firm retires its stock and is completely financed by debt. All informed traders will trade bonds and \(R^B(F = m + e) = R^S(F = 0)\).

Note that when it is costly to acquire information, an increase in leverage not only affects the allocation of informed traders between the stock and bond markets but also affects the equilibrium number of informed traders. Proposition 2 describes the equilibrium number of informed traders and the aggregate informed trading profit as a function of leverage level \(F\).
Proposition 2. In equilibrium, the aggregate informed trading profit is $\pi(F) = \phi(F)c$, where the number of informed traders, $\phi(F)$, is as follows:

i) If $F \leq m - e$, then $\phi(F) = \left(\frac{z}{2q-1}\right) (m - F) \left(1 - \frac{c}{2q-1} \left(\frac{m-F}{e}\right)\right)$.

ii) If $m - e < F \leq F^*$, then $\phi(F) = \left(\frac{z}{2q-1}\right) \left(1 - \frac{c}{2q-1} \right) \left(\frac{m+e-F}{2}\right)$.

iii) If $F^* < F < m + e$, then $\phi(F) = \left(\frac{z}{2q-1}\right) \left(1 - \frac{c}{2q-1} \right) \left(\frac{m-e}{2}\right)$.

iv) If $F^* = m + e$, then $\phi(m + e) = \phi(0) = \left(\frac{z}{2q-1}\right) m \left(1 - \frac{c}{2q-1} \left(\frac{m}{e}\right)\right)$.

Here $\pi(F)$ increases in $F$ if $F < m - ((2q-1)e)/(2c)$, decreases in $F$ if $m - ((2q-1)e)/(2c) < F < F^*$, and then increases in $F$ if $F > F^*$.

Leverage has two opposite effects on the number of informed traders, and subsequently the aggregate informed trading profit. On the one hand, when the debt level is low, increasing $F$ removes the risk-free portion of the equity payoff. This makes equity more informationally sensitive. Since informed traders are wealth-constrained, they benefit from the improved information sensitivity of the equity and enjoy a higher trading profit. Hence, the sensitivity effect of leverage increases informed trading profit. On the other hand, since liquidity trading at date 1 is proportional to the amount of securities issued, increasing $F$ reduces the amount of liquidity trading associated with the firm’s equity and boosts competition among informed traders. Thus, the liquidity effect of leverage reduces informed trading profit.

The net effect of leverage depends on the relative magnitudes of the sensitivity and liquidity effects. Proposition 2 suggests that the sensitivity effect prevails when the debt level is low ($F < m - ((2q-1)e)/(2c)$). This is because, though equity has become more informationally sensitive, leverage has not caused a sufficiently large reduction of liquidity trading in the stock market. Overall, informed traders achieve higher profits as debt rises.

When the debt level rises beyond $m - ((2q-1)e)/(2c)$, increasing $F$ no longer significantly improves the informational sensitivity of the stock but continues to drive down the liquidity trading associated with the firm’s equity. The liquidity effect starts to dominate the sensitivity effect, and informed trading profit in the stock market declines.

Once $F$ passes $F^*$, however, some traders gravitate toward the debt market (Proposition 1). Competition among informed traders in the stock market is reduced. Informed traders as a group can exploit the liquidity trading in both the stock and bond markets. As a result, the aggregate informed trading profit rises.

\[13\] Here we closely follow Boot and Thakor’s (1993) definition of “information sensitivity” as “the percentage divergence between the ‘true’ value of the security and its value based on the prior assessment of the uniformed.” The sensitivity effect is empirically confirmed by Lesmond et al. (2008).
V. The Optimal Capital Structure

In the previous section we analyzed the liquidity premium associated with debt and equity. We now examine the operating efficiency gain associated with the informativeness of the security prices. We then derive the optimal capital structure based on the trade-off between the liquidity premium and the operating efficiency gain.

A. Informational Benefit of Debt

When traders acquire information, information revelation in the security markets can lead to a more efficient investment decision. The following proposition describes the operating efficiency gain arising from the informativeness of the securities prices as a function of $F$.

Proposition 3. The expected operating efficiency gain $G(F)$ is given by

$$G(F) = (L - (m - e)) \left(1 - \left(\frac{c}{2q-1}\right) \left(\frac{\max(m-F,e)}{e}\right)^2\right).$$

Proposition 3 implies that a firm can improve its operation by inferring useful information from its security price(s). The first term of equation (3), $L - (m - e)$, captures the magnitude of the operating efficiency gain; the firm liquidates instead of continuing to invest if the bad state $-e$ should occur. The second term of equation (3) is the probability that such a gain occurs. This probability depends on certain characteristics of the firm: the degree of leverage, the cost of information acquisition, the precision of the signal that an informed trader receives ($q$), and/or the information advantage of informed traders ($e$). In particular, achieving the operating efficiency gain is more likely as $F$ rises for the following reason: Increasing leverage (up until $m - e$) strips away the risk-free component of the cash flow, making equity more informationally sensitive. This encourages traders to acquire information, which in turn leads to a more informative stock price and thus a more efficient investment decision. When $F$ exceeds the risk-free threshold $m - e$, however, increasing leverage no longer leads to a higher probability of achieving the efficiency gain. Instead, the probability is only determined by the cost of information acquisition and the precision of the signal that an informed trader receives.

B. Choosing a Capital Structure

The date 0 expected value of the firm is

$$V(F) = m + G(F) - \pi(F).$$

\[14\] In a more general setting, this probability is likely to increase with $F$ even beyond $m - e$. This is the case, for example, in an earlier version of our model where $c$ is assumed to be 0 and the number of informed traders is exogenously given.
Lemma 2. For $F \leq m - ((2q - 1)/(2c))(L - (m - e))/(2z + e)$, $V(F)$ is maximized at $F^\circ = m - ((2q - 1)/c)e$, where $F^\circ < 0$. For $m - ((2q - 1)/(2c))(L - (m - e))/(2z + e) < F < m + e$, $V(F)$ is maximized at $F^\ast$.

Lemma 2 indicates that there are two local maximizers for the date 0 value of the firm. One is $F^\ast > 0$ and the other is $F^\circ < 0$. A negative $F$ can be interpreted as holding cash reserves in bank deposits or in government securities.

Consider first $F \leq m - ((2q - 1)/(2c))(L - (m - e))/(2z + e)$. In this lower level debt region, raising $F$ increases the liquidity premium (Proposition 2) but does not generate sufficient gain for the firm. This is because when $F$ is small, there is still a large volume of liquidity trading in the stock market, preventing the stock price from being informative enough to generate sufficient operating efficiency gain. Hence, the liquidity premium dominates the operating efficiency benefit; the overall value of the firm decreases as $F$ increases. To achieve the maximum value of the firm, the capital structure is chosen to minimize the informed trading profit. In fact, if $F$ decreases into the negative region, informed trading profit can be reduced further. At $F^\circ$, equity becomes so “safe” and its sensitivity to information is so low that informed traders have no incentive to acquire information. Informed trading profit becomes 0 and $V(F^\circ) = m$.\(^\text{15}\)

Next, consider $F > m - ((2q - 1)/(2c))(L - (m - e))/(2z + e)$. In this higher level debt region, the gain from information revelation, $G(F)$, continues to rise as $F$ increases, while the informed trading profit eventually decreases as higher $F$ drives down the liquidity associated with equity. At $F^\ast$, all the informed traders still trade equity (Proposition 1), while the supply of liquidity trading in the stock market reduces to the minimum. Since the competition among traders is at its keenest, the stock price conveys the most information and the aggregate informed trading profit reaches its minimum.

If the debt level were raised beyond $F^\ast$, some traders would gravitate toward the bond market so that informed traders, as a group, can exploit the liquidity trading in both stock and bond markets. This increases the aggregate informed trading profit (see Proposition 2). In addition, traders’ information would be “garbled” by more liquidity trading, which weakly decreases the degree of information revelation in the security markets and the operating efficiency gain $G(F)$.\(^\text{16}\)

Consequently, the firm’s value is maximized at $F^\ast$.

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\(^\text{15}\)In reality, there are always traders who may obtain information with low or no cost. In this case, there will be some informed trading.

\(^\text{16}\)In a more general setting where some informed traders can obtain information at no cost, the efficiency gain $G(F)$ is strictly decreasing in $F$, as we have shown in the previous version of the paper.
Figure 3 summarizes how the traders’ incentive to acquire information and trade between the two markets varies with $F$. As illustrated in Figure 3, $F^o$ is the highest debt level at which informed stock trading is not profitable. By contrast, $F^*$ is the highest debt level at which informed bond trading is not profitable. To determine the optimal capital structure, we compare the value of the firm at these two local maximizers, $V(F^o)$ and $V(F^*)$. Proposition 4 describes the results.

**Proposition 4.** The optimal debt level is $F^o$ if $m + e - (\frac{(L - (m - e))}{c})(\frac{(2q - 1)}{z}) > F^*$, and is $F^*$ otherwise.

The choice of capital structure depends crucially on both the operating efficiency improvement due to information revelation in the security markets and the liquidity premium due to the presence of informed trading. If the liquidity premium dominates the operating efficiency gain, then $V(F^o) > V(F^*)$. Proposition 4 suggests that the firm chooses $F^o$ as the optimal capital structure solely to reduce the informed trading profit. If the operating efficiency gain exceeds the liquidity premium, then $V(F^o) < V(F^*)$, and $F^*$ emerges as the optimal capital structure to facilitate information revelation at the lowest liquidity cost. In the next section, we describe in detail how the choice of $F^o$ and $F^*$ depends on the characteristics of the firm.

Our model thus suggests that any liquidity premium associated with the degree of leverage is costly to the firm. Differing from the standard financial distress costs, this type of cost arises from the gain or loss of information. Consequently, it cannot be mitigated through debt renegotiation, as the information loss cannot be recovered by restructuring debt when default occurs.

Researchers and practitioners have long cast doubts about the significance of financial distress costs and believed many firms to be underleveraged (Haugen and Senbet (1988)). Our results substantiate the cost of debt financing by suggesting that the cost of debt may be underestimated if we ignore the liquidity premium and/or operating efficiency loss associated with overleverage.

Interestingly, the majority of the existing factors affecting a firm’s capital structure decision, such as tax, bankruptcy cost, and agency issues, appear less likely to interact with the two determinants studied in our model. This suggests
that a firm may deviate from the optimum derived in Proposition 4 by choosing its capital structure based on factors other than informational efficiency and liquidity premium, but the cost of debt financing still prevails.

Note that our framework allows us to explicitly characterize the optimal level of capital structure, the value of the firm, and the operating gain arising from information revelation in the secondary markets. For example, Figure 4 presents the operating efficiency gain, the liquidity premium, and the value of the firm as a function of leverage.\(^{17}\) In this example, the optimal level of debt can be explicitly calculated as \(F^* = 62.36\) for the following parameters: \(m = 60, e = 30, L = 40, q = \frac{2}{3}, z = 0.5,\) and \(c = 0.1.\)

\[\begin{align*}
F^* &= 62.36 \\
F &= m + e \\
\end{align*}\]

**FIGURE 4**

Numerical Example: Operating Efficiency Gain, Liquidity Premium, and Value of the Firm as a Function of \(F\)

In Figure 4, the x-axis is the level of debt (\(F\)), and the y-axis is the dollar value of the operating efficiency gain, liquidity premium, and the value of the firm, respectively. Here \(m = 60, e = 30, L = 40, z = 0.5, q = \frac{2}{3},\) and \(c = 0.1.\)

Graph A. Operating Efficiency Gain and Liquidity Premium as a Function of \(F\)

Graph B. Value of the Firm as a Function of \(F\)

\(17\)Note that \(\pi(F)\) does not approach \(\pi(0)\) as \(F\) approaches \(m + e.\) Since the stock price is always more revealing than the bond price, liquidation caused by information revelation in the stock price reduces bond traders’ information advantage, which reduces the aggregate informed trading profit from both markets. This “cross-revelation” effect exists for any debt level \(F < m + e.\) At the limit \(F = m + e,\) the firm is completely financed with debt, so debt is the same as unleveraged equity. As illustrated in Figure 4, \(\pi(m + e) = \pi(0)\) and \(\pi(F)\) has a jump at \(F = m + e.\)
VI. Empirical Implications

In this section we discuss our theory’s empirical implications and provide some supporting empirical evidence.

A. Negative Debt $F^o$ as the Optimal Capital Structure

In our model, the liquidity premium associated with leverage is costly to the firm. This cost can be so substantial relative to the benefit of information revelation that a firm chooses zero or negative debt to avoid it. To illustrate the significance of this effect, consider the following simple numerical example: $m = 100$, $e = 90$, $L = 10$, $c = 0.2$, $z = 0.5$, and $q = \frac{2}{3}$. In this case, $F^o = -50$, $F^* = 40$, and $V(F^o) = 100 > V(F^*) = 91$. The value of the firm under negative debt is about 10% higher than when the debt is 44% of the firm value. If new information arrives and our one-period model is repeated for every period, the firm can use negative debt to reduce the liquidity premium equivalent to 10% of its value in each period.

Even in the presence of corporate tax, it is possible that liquidity premiums outweigh tax shield benefits. In the above example, in the absence of financial distress cost, a 40% tax rate for a firm with a 44% debt ratio and a 10% interest rate on its debt translates to a 1.8% value gain from tax advantage. Obviously, the liquidity cost associated with a higher level of debt dwarfs the tax benefit.

Proposition 5 describes when $F^o$ is more likely to be the optimal capital structure.

Proposition 5. It is more likely for $F^o$ to be optimal when $L$ is smaller and/or $z$ is larger.

When liquidation fails to yield a significantly higher value than continuation in the bad state ($L$ is closer to $m - e$), the benefit of information revelation in the secondary markets accrued to the firm becomes economically small. Proposition 5 suggests that in this case, the firm should eliminate the liquidity premium by choosing a capital structure that deters any incentive to acquire information. This generates the following empirical implication: Firms with better growth opportunities and/or less tangible assets (smaller $L$ relative to $m - e$) are more likely to have a negative debt by holding large cash reserves while retaining little or no debt.

Our results provide an explanation for the well-documented phenomenon that a significant fraction of firms consistently hold no debt or even hold large excess cash reserves. For example, Strebulaev and Yang (2006) find that between 1987 and 2003, 11% of large public nonfinancial U.S. firms showed no debt in their capital structure, and 36% had zero or negative net debt. Anecdotal evidence includes IBM and Kodak in the 1970s and Microsoft in the 1990s. These firms had no debt and held substantial levels of cash reserves, which allowed them to borrow at nearly risk-free rates and to capture the tax shield benefit. Instead, they forewent such tax benefits and appeared to be “negatively leveraged.” Consistent with our theory, Opler et al. (1999) show that firms with strong growth opportunities, firms operating in risky businesses, and small firms tend to hold dramatically more cash than other firms. Mikkelson and Partch (2003) also document that firms having
persistent and substantial holdings of cash tend to be small and fast-growing, have high R&D spending, and have little or zero debt.

Another implication of Proposition 5 is that firms are more likely to choose $F^o$ as optimal when the uncertainty surrounding the liquidity trading of their securities becomes higher (larger $z$). Since higher uncertainty in liquidity trading allows informed traders to exploit the uninformed to a greater extent, the liquidity premium is more likely to dominate the operating efficiency gain. In this case, $F^o$ becomes more attractive to these firms in their effort to diminish the high liquidity premium caused by high uncertainty in liquidity trading. Our model therefore predicts that, other things being equal, firms in countries whose security trading has a higher turnover rate are more likely to hold excess cash reserves and to have little or no debt.

**Proposition 6 (Negative Debt and Firm Characteristics).** Negative debt level $F^o$ increases in $c$ and $m$ and decreases in $e$.

Proposition 6 implies that the debt is less negative (i.e., the firm holds smaller cash reserves) if the cost of information acquisition is higher, the firm is more profitable, and/or the risk associated with the firm is lower. In our model, the firm chooses the negative debt level $F^o$ to make equity so safe that traders have no incentive to become informed. When the cost of information acquisition ($c$) decreases or the information advantage of an informed trader ($e$) increases, however, informed equity trading becomes more attractive. An even lower debt level (larger cash reserve) is needed to discourage any information acquisition effort. Note also that in our model, one interpretation of $e$ is the information advantage that informed traders have, which is measured relative to the mean value of the firm, $m$. For a given level of $e$, a higher level of $m$ makes the equity relatively “safer” (i.e., less sensitive to information). Consequently, it requires a lower level of excess cash reserves (a less negative $F^o$) to discourage traders from becoming informed.

**B. $F^*$ as the Optimal Capital Structure**

In our model, a firm only favors $F^*$ as the optimal capital structure if it anticipates that the expected operating efficiency gain exceeds the liquidity premium associated with informed trading. Proposition 7 describes the relationship between the optimal capital structure and a firm’s characteristics in this case.

**Proposition 7 (Leverage and Firm Characteristics).** Debt level $F^*$ increases in $m$ and $L$ and decreases in $e$.

Since one can interpret $F^o$ as the highest debt level at which informed stock trading is unprofitable and $F^*$ as the highest debt level at which informed bond trading is unprofitable, the intuition for this proposition is similar to that of Proposition 6. Informed bond trading is less attractive when $e$ decreases or $m$ increases, which reduces traders’ information advantage. Informed bond trading is also less profitable when the liquidation value $L$ is higher and therefore liquidation is more likely, in which case the downside information advantage is limited by $L$. Consequently, the liquidity trading associated with the stock can be reduced further by
an even higher debt level $F^*$ without fear that some of the informed traders will switch to the bond market.

Proposition 7 predicts that among firms with a positive amount of debt, those that are more profitable borrow more. It also predicts that firms with less risk or more tangible assets hold more debt. While this prediction is similar to the predictions of traditional trade-off theory, the underlying reasoning is different.

**Proposition 8 (Leverage and Cost of Information Acquisition).** Debt level $F^*$ increases in the cost of information acquisition $c$ and decreases in the information precision $q$.

For any given level of debt $F$, the high cost of information acquisition (or low precision of the signal) discourages traders from becoming informed (see Proposition 2). The subsequent decline in the number of informed traders in the stock market reduces competition and improves the individual trader’s profit. The firm is then able to raise the debt level further to reduce liquidity trading in the stock market and boost competition among informed traders.

Proposition 8 therefore implies that, other things being equal, firms with higher information acquisition costs borrow more and firms in general rely less on debt financing if their information environments improve over time. We are not aware of any empirical work directly testing these implications. However, they appear to be testable.

**Proposition 9.** At $F^*$, liquidity premium ($\pi$) increases in $e$ and decreases in $L$.

Greater information advantage (higher $e$) allows traders to obtain a higher informed trading profit, which ultimately leads to a larger liquidity premium. Greater $L$, on the other hand, reduces the informed trading profit from $S_1 - (m - e)$ to $S_1 - L$ per share traded by replacing the downside payoff of $m - e$ with $L$ in the case of liquidation.

Since $\pi = \phi c$, the number of informed traders can be determined in the context of a firm’s fundamentals. Proposition 9 suggests that firms with higher risk (higher $e$) and/or more intangible assets (lower $L$) attract more informed trading.

### C. Clustering around $F^*$ versus Clustering around $F^o$

In our model, the choice of optimal capital structure depends on the trade-off between lowering liquidity premiums and improving operating efficiency resulting from information revelation in capital markets. If information revelation does not translate into a significant increase in its operating efficiency, a firm holds negative debt in order to deter any incentive for informed trading. If, instead, information revelation dramatically improves its operating efficiency, a firm aims at a positive debt level to facilitate maximum information revelation at the lowest liquidity cost.

Our theory indicates that capital structure choices cluster around two possible debt levels: Firms that benefit little from information revelation in the secondary markets prefer holding excess cash reserves with no debt; firms that would benefit significantly from information revelation choose positive capital structure. Hence, the two levels of debt clustering suggest caution in drawing empirical predictions from our model.
To illustrate, consider the following example where \( e = 30, L = 40, q = \frac{2}{3}, z = 0.5, \) and \( c = 0.1. \) The relationship between a firm’s optimal debt level and \( m \) is presented in Figure 5. As shown in Figure 5, \( V(F^*) > V(F^o) \) for \( m < 66.294, \) and the optimal debt level is \( F^*. \) For \( m > 66.294, \) however, the optimal debt level becomes \( F^o. \) Although the optimal capital structure increases in \( m \) both before and after the threshold level of 66.294, it shifts from the positive level of \( F^* \) to the negative level of \( F^o \) once \( m \) surpasses 66.294.

**FIGURE 5**

**Numerical Example: Value of the Firm under \( F^o \) and \( F^* \) as a Function of \( m \)**

In Figure 5, the x-axis is the ex ante value of the firm (\( m \)), and the y-axis is the dollar value of the firm under \( F^o \) and \( F^* \), respectively. Here \( L = 40, e = 30, z = 0.5, q = \frac{2}{3}, c = 0.1, \) and \( m \) ranges from 50 to 70.

The previous example suggests that the prediction from our model—more profitable firms have higher levels of debt—applies only within its own cluster. Since firms in the \( F^o \) cluster tend to have a higher \( m \), the prediction can be the opposite across the two clusters (more profitable firms tend to be in the zero debt cluster). This implies that it may be necessary to separate firms of different clusters when testing our model’s implications.

**VII. Extensions**

**A. Private Placement of Corporate Debt**

One implication from our model is that the value loss due to the liquidity premium of a firm’s securities cannot be solved through a private placement of debt. This is because publicly traded debt allows investors who experience liquidity shocks to recover at least some value from bond trading, even if they have to bear a loss to informed traders. By placing debt privately, investors are forced to bear the entire loss due to the complete illiquidity of their investment.\(^{18}\) Therefore, in the absence of other functions such as monitoring performed by private debtholders, a firm prefers to issue public bonds instead of private (nontraded) debt.

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\(^{18}\)It should be pointed out that certain types of private debt, such as bank loans, are intermediated in such a way that the ultimate investors are still provided with liquidity (Diamond and Dybvig (1983)). However, this type of financing is associated with government intervention, which induces moral hazard problems. Comparison of this type of financing to public debt is beyond the scope of this paper.
debt as long as the level of liquidity trading is sufficiently high. This is consistent with the fact that large firms tend to issue more public debt. Small firms typically use more private debt because they need to be monitored more.

Another often-mentioned benefit of private debt is that it is easily renegotiable. The conventional view is that by mitigating the underinvestment problem of Myers (1977), the low cost of debt renegotiation leads to a significant advantage for private debt financing. In our model, however, overleverage increases the liquidity premium of a firm’s securities, a value loss that cannot be avoided even when debt renegotiation is costless. Therefore, our model has different implications for the cost significance of private debt financing.

B. When the Bond Market Makers Can Observe the Stock Price

In our one-period (static) model, the prices of the firm’s securities are set simultaneously. This is a simplifying assumption that does not affect the main message of our analysis. Since the firm’s operating decision is contingent on the information conveyed by the two securities’ prices and market makers rationally anticipate this, the prices of the two securities are dependent upon each other despite the fact that market makers in each market set the security price without observing the price in the other market.

The obverse of this assumption is that the two security markets open sequentially such that market makers in one market set their final price after observing the price in the other market. Because the stock price is always more informative in our model, market makers in the stock market never find it useful to observe the bond price. If, instead, the bond market opens after the stock market, then the bond price fully reflects the information conveyed by the stock price. As shown in Appendix III of the Internet version of the paper, informed traders will find it unprofitable to trade the bond. The firm chooses the debt level as close to \( m + e \) as possible to minimize the liquidity trading associated with its equity. Consequently, the stock price almost always reveals the true state.

The one-period static setting drives the result that informed bond trading is completely unprofitable no matter how high the debt level is relative to \( m + e \). In reality, new information arrives constantly and trades occur in each market continuously. Market makers may not be able to observe all information and the stock price before any trade takes place in the bond market. To fully incorporate, the real world would require a dynamic trading model.

Therefore, we believe that the simultaneous opening of the two security markets is more realistic within the context of the one-period model. In contrast, the sequential opening of bond and stock markets is more applicable to a multiperiod dynamic model. However, in a dynamic trading model, informed traders place their trade strategically over time, and market makers update their beliefs

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20In this respect, price setting within either of the two markets mimics a scenario that lies between the two opposite assumptions discussed above. Nevertheless, if the bond market makers can learn something (even though not everything) from the stock market before setting their own price, the debt level \( F^* \) will be higher than the one given in our propositions. The results concerning \( F^* \) are not affected.
and revise prices gradually. In addition, the firm’s investment decision is set dynamically. Such a model precludes a closed-form solution to the security market equilibrium.

C. When Liquidity Traders Can Wait

In our model, investors submit orders before they observe the price, so whether they see one or both prices makes no difference. We do not consider the scenario where some investors are able to wait and learn from the price in the other market, a dimension that generally requires a dynamic model to explore explicitly. However, without going into details, we attempt to discuss some specific cases here. For example, consider the case where there are two classes of liquidity traders, as in Admati and Pfleiderer (1988): the nondiscretionary liquidity traders who have to trade immediately and the discretionary liquidity traders who can wait. In our model, a continuum of informed traders compete as in the perfectly competitive market. At the margin, if an informed investor does not trade immediately, his information will be revealed through the trading of many other informed traders. Hence, his information advantage will be lost. As a result, both the informed traders and the nondiscretionary liquidity traders will trade immediately. On the other hand, the discretionary liquidity traders will indeed prefer to wait to learn from the prices in both markets and then trade at the prices that reflect the new information. In the spirit of this argument, the liquidity traders in our model should be interpreted as those who cannot wait.

VIII. Conclusions

This study examines how a firm’s capital structure policy affects the informational efficiency of its security prices. We show that the optimal capital structure is determined by trading off between improving the operating efficiency through information revelation in the firm’s security prices and lowering the liquidity premium demanded by average investors in the presence of informed trading. When information is less imperative for improving its operating decisions, a firm issues zero or negative debt by holding excess cash reserves to reduce socially wasteful information acquisition and the liquidity premium associated with it. When information is crucial to a firm’s operating decisions, the optimal debt level is one that achieves maximum information revelation at the lowest possible liquidity cost.

Our model can explain why many firms consistently hold no debt or even excess cash reserves. By exploring how capital structure policy affects liquidity premiums associated with debt and equity, it also establishes a theoretical link between firms’ financial decisions and their security returns. Our model provides a starting point for future research on more general topics, such as whether issuing securities other than debt and equity can mitigate the liquidity cost.

Appendix A. Notations

Appendix A provides the definitions of symbols used in the paper.

\( \theta \): the aggregate liquidity risk of date 0 investors, expressed as the fraction of selling investors in each of the two markets. \( \theta \) is uniformly distributed on \( [\bar{\theta} - z, \bar{\theta} + z] \).
L: the value of the firm in case of liquidation.

$m + \varepsilon$: the value of the firm without liquidation. The realization of $\varepsilon$ is either $e$ or $-e$ with equal probability.

$q$: the probability that the signal an informed investor receives correctly reveals the realization of $\varepsilon$. $q$ is assumed to be greater than $\frac{1}{2}$.

$\phi$: the measure of informed investors.

c: the cost of information acquisition for each informed investor.

$\tau$: denotes $\phi (2q - 1) / z$.

$\lambda^B$ and $\lambda^S$: the fraction of informed traders who trade the bond and the stock, respectively.

$R^S$ and $R^B$: the expected return for a (marginal) informed trader from trading the stock and the bond, respectively.

$\pi$: the aggregate informed trading profit from the two security markets.

$V$: the date 0 value of the firm.

$B_0, B_1, S_0, S_1$: the date 0, date 1 value of the bond and stock, respectively.

$G(F)$: the expected efficiency gain of the firm due to the information conveyed in the security price.

**Appendix B. Deriving Stock and Bond Prices**

A date 1 investor who acquires information at cost $c$ receives a noisy signal about $\varepsilon$. That is, if the true state is $\varepsilon = e$, then with probability $q$ the signal correctly reveals $e$, and with probability $1 - q$ the signal incorrectly reveals the state as being $-e$. However, if the true state is $-e$, then with probability $q$ the signal correctly reveals $-e$, and with probability $1 - q$ the signal reveals $e$.

1. **The Stock Price**

When the true state is $e$, with probability $q$, the signal received by an informed trader reveals $e$ and the trader submits a buy order; with probability $1 - q$, the signal reveals $-e$ and the trader submits a sell order. Note that a buy order can be written as a negative sell order. Since each informed trader is endowed with one dollar, the measure of informed traders is $\phi$, and the fraction of informed traders who trade the stock is $\lambda^S$, we can write the expected total sell order (in dollar value) submitted by informed traders in the stock market as

$$-\phi \lambda^S q + \phi \lambda^S (1 - q) = -\phi (2q - 1) \lambda^S.$$  

The liquidity sell order (in dollar value) is $\theta S_1$. Thus, the aggregate dollar value of order flow from both liquidity traders and informed traders in the stock market is

$$\theta S_1 - \phi (2q - 1) \lambda^S.$$  

When the true state is $-e$, with probability $q$, the signal reveals $-e$ and the trader submits a sell order; with probability $1 - q$, the signal reveals $e$ and the trader submits a buy order. We then derive the (dollar value) sell order submitted by informed traders, $\phi (2q - 1) \lambda^S$, and the aggregate order flow in the stock market,

$$\theta S_1 + \phi (2q - 1) \lambda^S.$$
With \( \theta \) being uniformly distributed between \( \overline{\theta} - z \) and \( \overline{\theta} + z \), (B-1) satisfies

\[
(B-3) \quad (\overline{\theta} - z) S_1 - \phi (2q - 1) \lambda^S \leq \theta S_1 - \phi (2q - 1) \lambda^S \leq (\overline{\theta} + z) S_1 - \phi (2q - 1) \lambda^S,
\]

and (B-2) satisfies

\[
(B-4) \quad (\overline{\theta} - z) S_1 + \phi (2q - 1) \lambda^S \leq \theta S_1 + \phi (2q - 1) \lambda^S \leq (\overline{\theta} + z) S_1 + \phi (2q - 1) \lambda^S.
\]

If the aggregate order flow fully reveals the true state of \( \varepsilon \), the date 1 stock price \( S_1 \) is the realized payoff to the shareholders, and an informed trader’s profit is 0. If the order flow is uninformative about \( \varepsilon \), which, as illustrated in Figure 2, occurs when the range of (B-3) overlaps with the range of (B-4), an informed trader earns a positive profit.

Let \( \theta^*_S \) and \( \theta^{**}_S \) be the end points of the overlapping portion of the intervals for \( \theta S_1 - \phi (2q - 1) \lambda^S \) and \( \theta S_1 + \phi (2q - 1) \lambda^S \), respectively, such that if \( \theta \in [\theta^*_S, \overline{\theta} + z] \) or \( \theta \in [\overline{\theta} - z, \theta^{**}_S] \), then the order flow is uninformative. From Figure 2, it is clear that:

i) If \( \theta \in [\overline{\theta} - z, \theta^*_S] \), then the order flow fully reveals \( \varepsilon = e \). The quoted price is \( S_1 = m + e - F \).

ii) If \( \theta \in (\theta^*_S, \overline{\theta} + z] \), then the order flow fully reveals \( \varepsilon = -e \); the firm liquidates.

Because of the limited liability of equity, market makers quote stock price as

\[
\frac{m + e - F + \max (m - e - F, 0)}{2}.
\]

We now derive \( \theta^*_S \) and \( \theta^{**}_S \).

Since \( \theta^*_S \) and \( \theta^{**}_S \) are the end points for the overlapping portion of intervals (B-3) and (B-4) (see Figure 2), they satisfy

\[
(B-6) \quad \theta^*_S S_1 - \phi (2q - 1) \lambda^S = (\overline{\theta} - z) S_1 + \phi (2q - 1) \lambda^S
\]

and

\[
(B-7) \quad (\overline{\theta} + z) S_1 - \phi (2q - 1) \lambda^S = \theta^{**}_S S_1 + \phi (2q - 1) \lambda^S.
\]

We then obtain

\[
(B-8) \quad \theta^*_S = \overline{\theta} - z + \frac{2 \phi (2q - 1) \lambda^S}{S_1} \quad \text{and} \quad \theta^{**}_S = 2 \overline{\theta} - \theta^*_S.
\]

Substituting (B-5) into (B-8),

\[
(B-9) \quad \theta^*_S = \overline{\theta} - z + \frac{4 \phi (2q - 1) \lambda^S}{m + e - F + \max (m - e - F, 0)}.
\]

In addition, (B-3) overlapping with (B-4) requires

\[
(\overline{\theta} + z) S_1 - \phi (2q - 1) \lambda^S - (\theta^*_S S_1 - \phi (2q - 1) \lambda^S)
\]

\[
= \theta^{**}_S S_1 + \phi (2q - 1) \lambda^S - ((\overline{\theta} - z) S_1 + \phi (2q - 1) \lambda^S).
\]

It is straightforward to verify that the above condition is satisfied with (B-8).
2. The Bond Price

We derive the bond price in the same way. Given that the fraction of informed traders who trade the bond is $\lambda^B$, and that the (dollar value of) liquidity sell order is $\theta B_1$, the aggregate dollar order flow from both liquidity traders and informed traders in the bond market is

$$\theta B_1 - \phi (2q - 1) \lambda^B$$

if the true state is $e$, and is

$$\theta B_1 + \phi (2q - 1) \lambda^B$$

if the true state is $-e$. With $\theta$ being uniformly distributed between $\bar{\theta} - z$ and $\bar{\theta} + z$, (B-10) satisfies

$$(\bar{\theta} - z) B_1 - \phi (2q - 1) \lambda^B \leq \theta B_1 - \phi (2q - 1) \lambda^B \leq (\bar{\theta} + z) B_1 - \phi (2q - 1) \lambda^B,$$

and (B-11) satisfies

$$(\bar{\theta} - z) B_1 + \phi (2q - 1) \lambda^B \leq \theta B_1 + \phi (2q - 1) \lambda^B \leq (\bar{\theta} + z) B_1 + \phi (2q - 1) \lambda^B.$$

The informed bond trading profit is positive whenever the bond market order flow is uninformative about $\varepsilon$, and it is 0 when the order flow fully reveals $\varepsilon$. Let $\theta^*_B$ and $\theta^*_B^*$ be the end points of the overlapping portion of the intervals for $\theta B_1 - \phi (2q - 1) \lambda^B$ and $\theta B_1 + \phi (2q - 1) \lambda^B$, respectively, such that if $\theta \in [\theta^*_B, \bar{\theta} + z]$ or $\theta \in [\bar{\theta} - z, \theta^*_B^*]$, then the order flow is uninformative. As illustrated in Figure 2, the stock price is more informative than the bond price. This indicates that in some cases the bond market order flow is uninformative, but the stock market order flow reveals the true state.22

i) If $\theta \in [\bar{\theta} - z, \theta^*_B^*]$, then the order flow fully reveals $\varepsilon = e$. The quoted bond price is $B_1 = F$. 

ii) If $\theta \in (\theta^*_B, \bar{\theta} + z]$, then the order flow fully reveals $\varepsilon = -e$; the firm liquidates. The payoff to the bondholder is $F$ if the liquidation value exceeds the face value of the debt. Otherwise, the payoff to the bondholder is $L$. Hence, the quoted price is $B_1 = \min (F, L)$. 

iii) If $\theta \in [\bar{\theta} - z, \theta^*_B^*]$ or $\theta \in [\theta^*_B, \theta^*_B^* + \theta^*_B - (\bar{\theta} - z)]$, then neither the bond market order flow nor the stock market order flow is informative about $\varepsilon$. The firm does not liquidate because $m > L$. If $\varepsilon = e$, then the payoff to the bondholders is $F$. If $\varepsilon = -e$, then the payoff is $\min (F, m - e)$. Since both states are equally likely, the quoted bond price becomes

$$B_1 = \frac{F + \min (F, m - e)}{2}.$$ 

iv) If $\theta \in (\theta^*_B^* + \theta^*_B - (\bar{\theta} - z), \bar{\theta} + z]$ or $\theta \in (\theta^*_B^*, \theta^*_B^*)$, then the bond market order flow is not informative, but the stock market order flow is. Even if the bond market makers cannot observe the true state from the bond market order flows, they anticipate that the firm will liquidate whenever the stock price reveals $\varepsilon = -e$. Therefore, the quoted bond price is

22For a formal proof, see Corollary 1 in Appendix C.
\[ B_1 = \frac{F}{2} + \min(F, L). \]

Similar to \( \theta^*_S \) and \( \theta^{**}_S \), we derive \( \theta^*_B \) and \( \theta^{**}_B \) as

\[ \theta^*_B = \bar{\theta} - z + \frac{2\phi (2q - 1) \lambda_B}{B_1} \quad \text{and} \quad \theta^{**}_B = 2\bar{\theta} - \theta^*_B. \]

Substituting (B-14) into (B-16),

\[ \theta^*_B = \bar{\theta} - z + \frac{4\phi (2q - 1) \lambda_B}{F + \min(F, m - e)}. \]

**Appendix C. Proofs**

**Proof of Lemma 1.** To derive the informed stock trading return, consider first that the noisy signal reveals \( e \). Since a trader can only trade shares up to his endowment of $1, he can buy \( \frac{1}{S_1} \) shares of the stock at price \( S_1 \). Note that the precision of the signal is \( q \), and that informed stock trading is profitable as long as the aggregate order flow in the stock market is not revealing. A trader’s return from buying the stock can be calculated as

\[ r^S(e) = q \int_{\theta^*_S}^{\bar{\theta} + z} \left( \frac{m + e - F}{S_1} - 1 \right) \frac{d\theta}{2z} + (1 - q) \int_{\bar{\theta} - z}^{\theta^*_S} \left( \max \left( \frac{m - e - F}{S_1}, 0 \right) - 1 \right) \frac{d\theta}{2z}. \]

Substituting \( \theta^{**}_S = 2\bar{\theta} - \theta^*_S \) (see (B-8)), the above expression becomes

\[ r^S(e) = \left( \frac{\bar{\theta} + z - \theta^*_S}{2z} \right) \left( q \left( \frac{m + e - F}{S_1} - 1 \right) + (1 - q) \left( \max \left( \frac{m - e - F}{S_1}, 0 \right) - 1 \right) \right). \]

If, instead, the noisy signal reveals \(-e\), the trader sells \( \frac{1}{S_1} \) shares of the stock at price \( S_1 \). His return from selling the stock is calculated as

\[ r^S(-e) = q \int_{\bar{\theta} - z}^{\theta^*_S} \left( 1 - \max \left( \frac{m - e - F}{S_1}, 0 \right) \right) \frac{d\theta}{2z} + (1 - q) \int_{\theta^*_S}^{\bar{\theta} + z} \left( 1 - \frac{m + e - F}{S_1} \right) \frac{d\theta}{2z} = \left( \frac{\bar{\theta} + z - \theta^*_S}{2z} \right) \left( q \left( 1 - \max \left( \frac{m - e - F}{S_1}, 0 \right) \right) + (1 - q) \left( 1 - \frac{m + e - F}{S_1} \right) \right). \]

The ex ante probability that an informed trader receives a particular signal is \( \frac{1}{2} \). So the expected informed stock trading return is

\[ R^S = \frac{r^S(e)}{2} + \frac{r^S(-e)}{2} = \left( \frac{2q - 1}{2} \right) \left( \frac{\bar{\theta} + z - \theta^*_S}{2z} \right) \left( \frac{m + e - F}{S_1} - \max \left( \frac{m - e - F}{S_1}, 0 \right) \right). \]
Plugging \( \theta_s^* = \bar{\theta} - z + (2\phi (2 q - 1) \lambda^S)/S_1 \) (see (B-8)) and (B-5) into (C-3), and denoting \( \tau \equiv \phi (2 q - 1)/z \), we obtain

\[
(C-4) \quad R^S = \left( \frac{2q - 1}{2} \right) \left( 1 - \frac{2 \tau \lambda^S}{m + e - F + \max (m - e - F, 0)} \right) \times \left( \frac{m + e - F - \max (m - e - F, 0)}{m + e - F + \max (m - e - F, 0)} \right).
\]

When \( F \leq m - e, \max (m - e - F, 0) = m - e - F \), so

\[
(C-5) \quad R^S = (2q - 1) \left( 1 - \frac{\tau \lambda^S}{m - F} \right) \left( \frac{e}{m - F} \right).
\]

When \( F > m - e, \max (m - e - F, 0) = 0 \), so

\[
(C-6) \quad R^S = (2q - 1) \left( 1 - \frac{2 \tau \lambda^S}{m + e - F} \right).
\]

From (C-5) and (C-6), an informed stock trader’s expected return \( R^S \) can be written as

\[
(C-7) \quad R^S = (2q - 1) \left( 1 - \frac{2 \tau \lambda^S}{m + e + \max (m - e, e)} \right) \left( \frac{e}{\max (m - e, e)} \right).
\]

To derive the informed bond trading return, consider first that the noisy signal that a trader receives reveals \( e \). The trader can buy \( 1/B_1 \) shares of the corporate bond at price \( B_1 \). While informed trading is profitable as long as the aggregate order flow in the bond market is not revealing (see interval \( [\theta_{\bar{B}}^*, \bar{\theta} + z] \) for \( e = e \) and interval \( [\bar{\theta} - z, \theta_{\bar{B}}^*] \) for \( e = -e \) in Figure 2), this return is affected by whether or not the stock price is informative. Given that the precision of the signal is \( q \), the trader’s return from buying the bond is

\[
\rho^B (e) = q \left( \int_{\theta_{\bar{B}}^* + z}^{\theta_s^* + \theta_{\bar{B}}^* - (\bar{\theta} - z)} \frac{F}{F_{\max (F, m - e)}} \left( \frac{e}{m - F} \right) d\theta \right) - 1 \frac{e}{2z}
\]

\[
\quad + \int_{\theta_{\bar{B}}^* + z}^{\theta_s^* + \theta_{\bar{B}}^* - (\bar{\theta} - z)} \left( \frac{F}{F_{\max (F, m - e)}} - 1 \right) \frac{d\theta}{2z}
\]

\[
\quad + (1 - q) \left( \int_{\theta_{\bar{B}}^* + z}^{\theta_s^* + \theta_{\bar{B}}^* - (\bar{\theta} - z)} \frac{\min (F, m - e)}{F_{\max (F, m - e)}} - 1 \right) \frac{d\theta}{2z}
\]

\[
\quad + \int_{\theta_s^* + \theta_{\bar{B}}^* - (\bar{\theta} - z)}^{\theta_{\bar{B}}^* - (\bar{\theta} - z)} \left( \frac{\min (F, L)}{F_{\max (F, m - e)}} - 1 \right) \frac{d\theta}{2z}.
\]

Since \( \theta_{\bar{B}}^* = 2\bar{\theta} - \theta_s^* \) and \( \theta_{\bar{B}}^* = 2\bar{\theta} - \theta_{\bar{s}}^* \), we have

\[
(C-8) \quad \bar{\theta} + z - (\theta_{\bar{B}}^* + \theta_{\bar{s}}^* - (\bar{\theta} - z)) = \theta_s^* - \theta_{\bar{s}}^* = \theta_s^* - \theta_{\bar{s}}^*.
\]

Then \( \rho^B (e) \) can be simplified as

\[
(C-9) \quad \rho^B (e) = \left( \frac{2q - 1}{2z} \right) \left( \theta_{\bar{s}}^* - (\bar{\theta} - z) \right) \left( \frac{F - \min (F, m - e)}{F + \max (F, m - e)} \right)
\]

\[
\quad + (\theta_{\bar{s}}^* - \theta_{\bar{s}}^*) \left( \frac{F - \min (F, L)}{F + \max (F, L)} \right).
\]
If, instead, the noisy signal reveals \(-e\), the trader sells \(1/B_1\) shares of the corporate bond at price \(B_1\). We calculate the trader’s return from selling the bond as

\[
r^B(-e) = q \left( \int_{\theta_B^c}^{\theta_s^*} \left( 1 - \frac{\min(F, m - e)}{F + \min(F, m - e)} \right) \frac{d\theta}{2z} + \int_{\theta_B^c}^{\theta_s^*} \left( 1 - \frac{\min(F, L)}{F + \min(F, L)} \right) \frac{d\theta}{2z} \right) + (1 - q) \left( \int_{\theta_B^c}^{\theta_s^* + \theta_B^* - (\theta - z)} \left( 1 - \frac{F}{F + \min(F, m - e)} \right) \frac{d\theta}{2z} \right) + \left( \theta_s^* - \theta_B^* \right) \left( F - \min(F, m - e) \right) \left( F + \min(F, m - e) \right).
\]

The ex ante probability that an informed trader receives a particular signal is \(\frac{1}{2}\). So the expected informed bond trading return is

\[
R^B = \frac{r^B(e)}{2} + \frac{r^B(-e)}{2} = \left( \frac{2q - 1}{2z} \right) \left( \theta_s^* - (\theta - z) \right) \left( F - \min(F, m - e) \right) \left( F + \min(F, m - e) \right) + \left( \theta_s^* - \theta_B^* \right) \left( F - \min(F, m - e) \right) \left( F + \min(F, m - e) \right).
\]

Substituting (B-5) into \(\theta_s^*\) (see (B-8)) and (B-14) into \(\theta_B^*\) (see (B-17)),

\[
\theta_s^* - (\theta - z) = 2z - \frac{4\phi(2q - 1)\lambda^s}{m + e - F + \max(m - e - F, 0)}
\]

and

\[
\theta_s^* - \theta_B^* = \frac{4\phi(2q - 1)\lambda^s}{m + e - F + \max(m - e - F, 0)} - \frac{4\phi(2q - 1)\lambda^B}{F + \min(F, m - e)}.
\]

The expected informed bond trading return is simplified as follows:
When \(F \leq m - e\),

\[
R^B = 0.
\]

When \(m - e < F < L\),

\[
R^B = (2q - 1) \left( 1 - \frac{2\tau\lambda^S}{m + e - F} \right) \left( \frac{F - (m - e)}{F + m - e} \right).
\]

Lastly, when \(F > L\),

\[
R^B = (2q - 1) \left( 1 - \frac{2\tau\lambda^S}{m + e - F} \right) \left( \frac{F - (m - e)}{F + m - e} \right) + \left( \frac{2\tau\lambda^S}{m + e - F} - \frac{2\tau\lambda^B}{F + m - e} \right) \left( \frac{F - L}{F + L} \right).
\]
Therefore, from (C-12), (C-13), and (C-14), \( R^B \) can be written as

\[
R^B = (2q - 1) \left( \frac{F - \min (F, m - e)}{F + \min (F, m - e)} \right) - \left( \frac{2\tau (2q - 1)}{m + e - F} \right) \left( \frac{F - \min (F, L)}{F + \min (F, L)} \right) \lambda^S - \left( \frac{2\tau (2q - 1)}{F + \min (F, m - e)} \right) \left( \frac{F - \min (F, L)}{F + \min (F, L)} \right) \lambda^B.
\]

This completes the proof. \( \square \)

**Proof of Proposition 1.** We consider, in order, the three regions for the debt level \( F \) in Proposition 1.

**Region 1** \((F \leq m - e)\). See that \( R^S (F) = (2q - 1)(1 - (\tau \lambda^S)/(m - F))(e/(m - F)) \) and \( R^B (F) = 0 \) (see (C-5) and (C-12) of the proof of Lemma 1), where \( \tau \equiv \phi (2q - 1)/z \).

Since each informed trader is endowed with one dollar, the profits from trading the stock and the bond are \$1 \times R^S (F) \) and \$1 \times R^B (F) \), respectively. In equilibrium we have

\[
(C-15) \quad (2q - 1) \left( 1 - \frac{\tau \lambda^S}{m - F} \right) \left( \frac{e}{m - F} \right) = c > 0.
\]

Solving \( \phi \) from (C-15),

\[
(C-16) \quad \phi = \left( \frac{z}{2q - 1} \right) \left( \frac{m - F}{\lambda^S} \right) \left( 1 - \frac{c}{2q - 1} \left( \frac{m - F}{e} \right) \right).
\]

Since \( \lambda^S \leq 1 \) and \( c < (2q - 1)e/m \) (Assumption 1), we can verify that \( \phi \) is positive:

\[
\phi > \left( \frac{z}{2q - 1} \right) \left( m - F \right) \left( \frac{1}{m - F} - \frac{c}{2q - 1} \left( \frac{m - F}{e} \right) \right) > 0.
\]

Note that (C-15) indicates that no informed traders trade the bond: \( \lambda^S = 1 \) and \( \lambda^B = 0 \). The equilibrium number of informed traders (C-16) becomes

\[
(C-17) \quad \phi = \left( \frac{z}{2q - 1} \right) (m - F) \left( 1 - \frac{c}{2q - 1} \left( \frac{m - F}{e} \right) \right).
\]

**Region 2** \((m - e < F \leq L)\). From (C-6) and (C-13) of the proof of Lemma 1, \( R^S = (2q - 1) \left( 1 - (2\tau \lambda^S)/(m + e - F) \right) \) and \( R^B = (2q - 1) \left( 1 - (2\tau \lambda^S)/(m + e - F) \right) \left( (F - (m - e))/(F + m - e) \right) \). Given that each trader can trade only up to his one dollar endowment, and that the cost of information acquisition is \( c \), in equilibrium, an informed trader’s profit satisfies

\[
(C-18) \quad (2q - 1) \left( 1 - \frac{2\tau \lambda^S}{m + e - F} \right) = c.
\]

Since \( \lambda^S \leq 1 \), \( m - e < F < L \), \( m > e \), and \( c < ((2q - 1)e)/m \) (Assumption 1), the equilibrium number of informed traders from (C-18), \( \phi = (z/(2q - 1)) \left( 1 - (c/(2q - 1)) \right) \left( (m + e - F)/(2\lambda^S) \right) \), is positive:

\[
\phi > \left( \frac{z}{2q - 1} \right) \left( 1 - \frac{c}{2q - 1} \right) \left( m + e - L \right) \left( 2 \right) > 0.
\]
Then \( (F - (m - \epsilon))/(F + m - \epsilon) < 1 \) implies that \( R^S > R^B \). All the informed traders still trade stocks: \( \lambda^S = 1 \) and \( \lambda^B = 0 \). The equilibrium number of informed traders becomes

\[(C-19) \quad \phi = \left( \frac{z}{2q-1} \right) \left( 1 - \frac{c}{2q-1} \right) \left( \frac{m + e - F}{2} \right).
\]

**Region 3 (\( F > L \)).** From (C-6) and (C-14) of the proof of Lemma 1, \( R^S = (2q - 1) \((1 - (2\tau \lambda^S)/(m + e - F)) \) and \( R^B = (2q - 1) \((1 - (2\tau \lambda^B)/(m + e - F)) \)/(F + m - \epsilon) + ((2\tau \lambda^S)/(m + e - F) - (2\tau \lambda^B)/(F + m - \epsilon))((F - L)/(F + L)) \). In this case, an informed trader is indifferent between trading the stock and trading the bond. In equilibrium, his informed trading profit is equal to his cost of information acquisition. So

\[(C-20) \quad (2q - 1) \left( 1 - \frac{2\tau \lambda^S}{m + e - F} \right) = c \quad \text{and}
\]

\[(C-21) \quad (2q - 1) \left( \left( 1 - \frac{2\tau \lambda^S}{m + e - F} \right) \left( \frac{F - (m - \epsilon)}{F + m - \epsilon} \right) \right. + \left. \frac{2\tau \lambda^S}{m + e - F} - \frac{2\tau \lambda^B}{F + m - \epsilon} \left( \frac{F - L}{F + L} \right) \right) = c,
\]

and \( \lambda^B = 1 - \lambda^S \).

From (C-20) we solve

\[(C-22) \quad 2\tau \lambda^S = \left( \frac{1 - \frac{c}{2q-1}}{m + e - F} \right).
\]

Plugging (C-22) and \( \lambda^B = 1 - \lambda^S \) into (C-21), (C-21) becomes

\[(C-23) \quad F - L \left( \frac{1 - \frac{c}{2q-1}}{F + m - \epsilon} \right) + \frac{F - L}{F + L} \left( \frac{1 - \frac{c}{2q-1}}{m + e - F} \left( \frac{c}{2q-1} \right) \right) = \frac{c}{2q-1}.
\]

We then solve \( \tau \) from (C-23):

\[(C-24) \quad \tau = \left( \frac{1 - \frac{c}{2q-1}}{m} - \left( \frac{c}{2q-1} \right) \right) m - \left( \frac{c}{2q-1} \right) \left( \frac{F + L}{F - L} \right).
\]

Substituting (C-24) into (C-22), we obtain the equilibrium fraction of informed traders who trade the stock,

\[(C-25) \quad \lambda^* = \left( \frac{1 - \frac{c}{2q-1}}{m} - \left( \frac{c}{2q-1} \right) \left( \frac{m + e - F}{2} \left( \frac{1 - c}{2q-1} \right) \right) \right).
\]

Then, \( \lambda^* \leq 1 \) implies

\[(C-26) \quad F^2 + \left( \frac{2q - 1 - 3c}{2q - 1 - c} \left( m - e \right) - L \right) F - L \left( m - e \right) \left( \frac{2q - 1 + c}{2q - 1 - c} \right) \geq 0.
\]

\( F^* \) is a root of (C-26) given by

\[(C-27) \quad F^* = \frac{-\alpha + \sqrt{\alpha^2 - 4\beta}}{2},
\]
where \( \alpha \equiv ((2q - 1 - 3c)/(2q - 1 - c))(m - e) - L \) and \( \beta \equiv -L(m - e)((2q - 1 + c)/(2q - 1 - c)) \). It is also straightforward to verify that \( F^* > L \).

Note that \( \alpha < 0 \) and \( \beta < 0 \). The other root of (C-26) is negative and can be ignored. Since the left-hand side of (C-26) is convex in \( F \), the inequality of (C-26) is satisfied for \( F > F^* \).

The inequality of (C-26) does not hold if \( F \leq F^* \), which implies \( \lambda^* \geq 1 \). Then, traders’ wealth constraint requires \( \lambda^* = 1 \). That is, the return on trading the stock is still high enough that all the informed traders prefer trading the stock.

To summarize, for \( L < F \leq F^* \), all informed traders trade stock: \( \lambda^L = 1 \) and \( \lambda^B = 0 \). When \( F > F^* \), \( \lambda^* < 1 \) fraction of informed traders trade the stock and \( 1 - \lambda^* \) fraction of traders trade the bond.

We now verify that \( \phi \) is positive in this region.

For \( L < F \leq F^* \), since \( \lambda^L(F) = 1, \tau = \phi((2q - 1)/z) \), (C-22) becomes \( \phi = c/(2q - 1))((m + e - F)/2) \). From Assumption 1, \( c < (2q - 1)e/m < 2q - 1 \), so \( \phi > 0 \). In particular, \( \phi(F^*) > 0 \).

For \( F > F^* \), \( \phi \) is given by (C-24):

\[
(C-28) \quad \phi = \left( \frac{z}{2q - 1} \right) \left( 1 - \frac{c}{2q - 1} \right) m - \left( \frac{c}{2q - 1} \right)(m - e) \frac{F + L}{F - L}.
\]

It is easy to verify that \( \phi'(F) > 0 \). Then \( \phi(F^*) > 0 \) implies \( \phi(F) > 0 \) for all \( F > F^* \). This completes the proof. \( \Box \)

**Corollary 1.** The stock price is more informative than the bond price.

**Proof.** We first show that in equilibrium, informed trading returns and the allocation of informed traders are consistent with the stock price being more informative than the bond price. We then establish that the opposite—the bond price is more informative than the stock price—cannot be true in equilibrium.

As illustrated in Figure 2, the stock price being more informative than the bond price requires \( \theta^*_B < \theta^*_S \) and \( \theta^*_B^* > \theta^*_S^* \), which are equivalent to \( \lambda^B/\lambda^S < B_1/S_1 \). Substituting \( B_1 \) and \( S_1 \),

\[
(C-29) \quad \frac{\lambda^B}{\lambda^S} < \frac{F + \min(F, m - e)}{m + e - F + \max(m - e - F, 0)}.
\]

When \( F \leq F^* \), \( \lambda^B = 0 \), and \( \lambda^S = 1 \), the inequality holds. When \( F > F^* \), to verify (C-29) is equivalent to verifying

\[
\frac{1 - \lambda^*}{\lambda^*} < \frac{F + m - e}{m + e - F},
\]

where

\[
\lambda^* = \left( 1 - \frac{c}{2q - 1} \right) \frac{m + e - F}{2 \left( \frac{c}{2q - 1} \right)(m - e) \frac{F + L}{F - L}}.
\]

This inequality can be simplified as

\[
-\frac{c}{2q - 1 - c} (m - e) \frac{F + L}{F - L} < 0,
\]

which always holds by Assumption 1. Therefore, (C-29) is satisfied in equilibrium.

Next, we prove by contradiction that the bond price cannot be more informative than the stock price.

Again, let \( \theta^*_B, \theta^*_B^*, \theta^*_S \), and \( \theta^*_S^* \) be such that if \( \theta \in [\theta^*_B, \theta + z] \) or \( \theta \in [\theta^*_S, \theta + z] \), then the bond market order flow is uninformative. If \( \theta \in [\theta^*_B^*, \theta + z] \) or \( \theta \in [\theta^*_S^*, \theta + z] \),
then the stock market order flow is uninformative. Suppose that the bond price is more informative than the stock price. This implies that \( \theta_B^* > \theta_S^* \) and \( \theta_B^{**} < \theta_S^{**} \).

If \( \theta \in [\bar{\theta} - z, \theta_B^*] \), then the bond market order flow fully reveals \( \varepsilon = e \), and the quoted price is \( B_1 = F \). If \( \theta \in (\theta_B^{**}, \bar{\theta} + z] \), then the bond market order flow fully reveals \( \varepsilon = -e \) and the firm liquidates. Market makers quote \( B_1 = \min (L, F) \). If \( \theta \in [\bar{\theta}, \bar{\theta} + z] \) or \( \theta \in [\bar{\theta} - z, \theta_B^{**}] \), then the bond market order flow is uninformative, and the quoted price is \( B_1 = (F + \min (F, m - e)) / 2 \).

If \( \theta \in [\bar{\theta} - z, \theta_S^*] \), then the stock order flow fully reveals \( \varepsilon = e \). The quoted stock price is \( S_1 = m + e - F \). If \( \theta \in (\theta_S^{**}, \bar{\theta} + z] \), then the order flow fully reveals \( \varepsilon = -e \) and the firm liquidates. The quoted price is \( S_1 = \max (L - F, 0) \). If \( \theta \in [\bar{\theta}, \bar{\theta} + z] \) or \( \theta \in (\theta_S^{**}, \theta_S^*) \), then both the market order flow and the stock market order flow are uninformative about \( \varepsilon \). The firm does not liquidate because \( m > L \). The quoted stock price is \( S_1 = (m + e - F + \max (m - e, F, 0)) / 2 \). If \( \theta \in (\theta_B^{**} + \theta_S^* - (\bar{\theta} - z), \bar{\theta} + z] \) or \( \theta \in (\theta_B^{**}, \theta_S^*) \), then the stock market order flow is informative, but the bond market order flow is informative. Even if market makers cannot observe the true state from the stock market order flows, they anticipate that the firm will liquidate whenever the bond price reveals \( \varepsilon = -e \). Therefore, the quoted stock price is \( S_1 = (m + e - F + \max (L - F, 0)) / 2 \).

Similar to Appendix B, it can be shown that \( \theta_B^*, \theta_S^*, \theta_S^{**} \), and \( \theta_B^{**} \) satisfy \( \theta_B^* = \bar{\theta} - z + (2\phi(2q - 1)\lambda^B)B_1, \theta_B^{**} = 2\bar{\theta} - \theta_B^*, \theta_S^* = \bar{\theta} - z + (2\phi(2q - 1)\lambda^S)S_1, \) and \( \theta_S^{**} = 2\bar{\theta} - \theta_S^* \), respectively.

We now derive the returns from informed bond trading and from informed stock trading separately.

The expected return to a trader from buying the bond when he receives a signal of \( e \) is

\[
r^B(e) = q \int_{\theta_B^*}^{\theta_B^{**}} \left( \frac{F}{\min (F, m - e)} - 1 \right) \frac{d\theta}{2z} + (1 - q) \int_{\theta_S^*}^{\theta_S^{**}} \left( \frac{\min (F, m - e)}{\min (F, m - e)} - 1 \right) \frac{d\theta}{2z}.
\]

The expected return to a trader from selling the bond when he receives a signal of \( -e \) is

\[
r^B(-e) = q \int_{\theta_S^*}^{\theta_B^{**}} \left( 1 - \frac{\min (F, m - e)}{\min (F, m - e)} \right) \frac{d\theta}{2z} + (1 - q) \int_{\theta_B^*}^{\theta_B^{**}} \left( 1 - \frac{F}{\min (F, m - e)} \right) \frac{d\theta}{2z}.
\]

The expected return to a bond trader \( R^B \) then becomes

\[
R^B = \frac{r^B(e)}{2} + \frac{r^B(-e)}{2} = \left( 2q - \frac{1}{2z} \right) \left( \theta_B^{**} - (\bar{\theta} - z) \right) \left( \frac{F - \min (F, m - e)}{F + \min (F, m - e)} \right).
\]

The expected return to a stock trader when he receives a signal of \( e \) is

\[
r^S(e) = q \int_{\theta_S^*}^{\theta_S^{**}} \left( \frac{m + e - F}{\max (m + e, F, 0)} - 1 \right) \frac{d\theta}{2z}
+ q \int_{\theta_B^*}^{\theta_B^{**}} \left( \frac{m + e - F}{\max (m + e, F, 0)} - 1 \right) \frac{d\theta}{2z}
+ (1 - q) \int_{\theta_S^*}^{\theta_S^{**}} \left( \frac{\max (m + e - F, 0)}{\max (m + e, F, 0)} - 1 \right) \frac{d\theta}{2z}
+ (1 - q) \int_{\theta_B^*}^{\theta_B^{**}} \left( \frac{\max (L - F, 0)}{\max (m + e, L - F, 0)} - 1 \right) \frac{d\theta}{2z}.
\]
The trader’s return from selling the stock when he receives a signal of \(-e\) can be calculated as
\[
R^S (-e) = q \int_{\theta - z}^{\theta_z^*} \left( 1 - \frac{\max (m - e - F, 0)}{m + e - F + \max (m - e - F, 0)} \right) \frac{d\theta}{2z} \\
+ q \int_{\theta_z^*}^{\theta_z^{**}} \left( 1 - \frac{\max (L - F, 0)}{m + e - F + \max (L - F, 0)} \right) \frac{d\theta}{2z} \\
+ (1 - q) \int_{\theta - z}^{\theta_z^* + \theta_z^* - (\bar{\theta} - z)} \left( 1 - \frac{m + e - F}{m + e - F + \max (m - e - F, 0)} \right) \frac{d\theta}{2z} \\
+ (1 - q) \int_{\theta_z^* + \theta_z^* - (\bar{\theta} - z)}^{\theta_z^{**} + \theta_z^* - (\bar{\theta} - z)} \left( 1 - \frac{m + e - F}{m + e - F + \max (L - F, 0)} \right) \frac{d\theta}{2z}.
\]

Then, the expected return to a stock trader,
\[
R^S = \frac{r^S (e)}{2} + \frac{r^S (-e)}{2},
\]
can be calculated as
\[
R^S = \left( \frac{2q - 1}{2z} \right) (\theta_{B^*} - (\bar{\theta} - z)) \left( \frac{m + e - F - \max (m - e - F, 0)}{m + e - F + \max (m - e - F, 0)} \right) \\
+ \left( \frac{2q - 1}{2z} \right) (\theta_{B^*} - \theta_z^*) \left( \frac{m + e - F - \max (L - F, 0)}{m + e - F + \max (L - F, 0)} \right).
\]

When \(F \leq m - e\), \(R^B = 0\). This implies \(\lambda^B = 0\): No informed traders will trade bonds, since \(c > 0\). When \(F > m - e\),
\[
(C-30) \quad R^B = \left( \frac{2q - 1}{2z} \right) (\theta_{B^*} - (\bar{\theta} - z)) \left( \frac{F - (m - e)}{F + (m - e)} \right)
\]
and
\[
(C-31) \quad R^S = \left( \frac{2q - 1}{2z} \right) (\theta_{B^*} - (\bar{\theta} - z)) \\
+ \left( \frac{2q - 1}{2z} \right) (\theta_{B^*} - \theta_z^*) \left( \frac{m + e - F - \max (L - F, 0)}{m + e - F + \max (L - F, 0)} \right).
\]

Since \((F - (m - e))/(F + (m - e)) < 1\), the first term on the right-hand side of \((C-31)\) is greater than \((C-30)\). The second term on the right-hand side of \((C-31)\) is positive when the bond price is more informative (i.e., \(\theta_{B^*} - \theta_z^* > 0\)). This implies that \(R^S > R^B\), which in turn implies that trading stock is more profitable and that \(\lambda^B = 0\).

The result that no informed traders will trade the bond \((\lambda^B = 0)\) is in contradiction with the assumption that the bond price is more informative. This completes the proof. \(\square\)

**Proof of Proposition 2.** In equilibrium, the aggregate informed trading profit is \(\pi = \phi c\). When \(F \leq m - e\),
\[
\phi (F) = \left( \frac{z}{2q - 1} \right) (m - F) \left( 1 - \frac{c}{2q - 1} \left( \frac{m - F}{e} \right) \right)
\]
(see \((C-17)\) in the proof of Proposition 1) is concave in \(F\). It is straightforward that \(F = m - ((2q - 1) e)/(2c)\) maximizes \(\phi (F)\). This implies \(\phi (F)\) increases in \(F\) for \(F < m - ((2q - 1) e)/(2c)\) and decreases in \(F\) for \(m - ((2q - 1) e)/(2c) < F \leq m - e\).
When \(m - e < F \leq F^*\),
\[
\phi (F) = \left(\frac{z}{2q - 1}\right) \left(1 - \frac{c}{2q - 1}\right) \left(\frac{m + e - F}{2}\right)
\]
(see (C-19) in the proof of Proposition 1). It decreases in \(F\).

When \(F^* < F < m + e\),
\[
\phi (F) = \left(\frac{z}{2q - 1}\right) \left(1 - \frac{c}{2q - 1}\right) m - \left(\frac{c}{2q - 1}\right) (m - e) \frac{F + L}{F - L}
\]
is given by (C-28). It clearly increases in \(F\).

Lastly, when \(F = m + e\), the firm is completely financed by debt (\(\lambda^S = 0\) and \(\lambda^B = 1\)), where debt is the same as equity. Now \(\pi (m + e) = \pi (0)\). Substituting \(F = 0\) into (C-17), we have
\[
\phi = \left(\frac{z}{2q - 1}\right) m \left(1 - \frac{c}{2q - 1}\right) \left(\frac{m}{e}\right).
\]
This completes the proof. \(\square\)

**Proof of Proposition 3.** The operating efficiency is positive when \(\theta \in [\theta_s^*, \bar{\theta} + z]\) for \(\varepsilon = -e\). In this case, the stock market’s order flow is fully revealing, so the firm liquidates instead of continuing its operation. Since \(\varepsilon = -e\) occurs with probability \(\frac{1}{2}\), the expected efficiency gain for the firm is
\[
G (F) = \frac{1}{2} \int_{\theta_s^*}^{\bar{\theta} + z} (L - (m - e)) \frac{d\theta}{2z}.
\]
Simplifying,
\[
(C-32) \quad G (F) = \frac{(L - (m - e)) (\bar{\theta} + z - \theta_s^*)}{4z} = \frac{\phi(2q-1) \lambda^S (L - (m - e))}{m + e - F + \max (m - e - F, 0)}.
\]
When \(F \leq m - e\), plugging (C-17) and \(\lambda^S = 1\) into (C-32),
\[
G (F) = \frac{1}{2} \left(1 - \frac{c}{2q - 1}\right) \left(\frac{m - F}{e}\right) (L - (m - e)) .
\]
When \(m - e < F \leq F^*\), plugging (C-19) and \(\lambda^S = 1\) into (C-32),
\[
G (F) = \frac{1}{2} \left(1 - \frac{c}{2q - 1}\right) (L - (m - e)) .
\]
When \(F > F^*\), \(\lambda^S = \lambda^*\). Equation (C-22) indicates that
\[
\tau \lambda^* = \left(1 - \frac{c}{2q - 1}\right) \frac{(m + e - F)}{2}.
\]
Then
\[
G (F) = \frac{1}{2} \left(1 - \frac{c}{2q - 1}\right) (L - (m - e)) .
\]
This completes the proof. \(\square\)
**Proof of Lemma 2.** The value of the firm is $V(F) = m + G(F) - \pi(F)$.  
For $F > F^*$, $G(F)$ is a constant, but $\pi(F)$ increases in $F$ (see Proposition 2). Therefore, $V(F^*) > V(F)$ for any $F > F^*$.

When $m - e < F \leq F^*$,

$$V(F) = m + \frac{1}{2} \left( 1 - \frac{c}{2q - 1} \right) (L - (m - e)) - \left( \frac{z}{2q - 1} \right) \left( 1 - \frac{c}{2q - 1} \right) \left( \frac{m + e - F}{2} \right) c,$$

which increases in $F$. Therefore, $V(F) > V(F^*)$ for any $F > F^*$.

When $F \leq m - e$,

$$V(F) = m + \frac{1}{2} \left( 1 - \frac{c}{2q - 1} \left( \frac{m - F}{e} \right) \right) (L - (m - e)) - \left( \frac{z}{2q - 1} \right) \left( m - F \right) \left( 1 - \frac{c}{2q - 1} \left( \frac{m - F}{e} \right) \right) c.$$  \hspace{1cm} (C-33)

The first-order condition of (C-33) is

$$\frac{1}{2} \left( \frac{c}{2q - 1} \right) \left( \frac{L - (m - e)}{e} \right) + \left( \frac{c}{2q - 1} \right) z - \frac{z}{2q - 1} \left( \frac{c^2}{2q - 1} \right) \left( \frac{2(m - F)}{e} \right) = 0,$$

in which

$$F = \frac{m - \left( \frac{2q - 1}{2c} \right) \left( \frac{L - (m - e)}{2z} \right) + e}{2c}.$$  \hspace{1cm} (C-34)

Note that $V(F)$ is convex as $(\partial^2 V(F)) / (\partial F^2) > 0$. Therefore, $V(F)$ increases in $F$ for

$$m - \left( \frac{2q - 1}{2c} \right) \left( \frac{L - (m - e)}{2z} \right) + e < F \leq m - e,$$

and decreases in $F$ for

$$F^o \leq F < m - \left( \frac{2q - 1}{2c} \right) \left( \frac{L - (m - e)}{2z} \right) + e,$$

where $F^o$ solves

$$\phi = \left( \frac{z}{2q - 1} \right) \left( m - F \right) \left( 1 - \frac{c}{2q - 1} \left( \frac{m - F}{e} \right) \right) = 0;$$

$\phi$ is given by (C-17). That is,

$$F^o = m - \frac{2q - 1}{c} e.$$  \hspace{1cm} (C-34)

Note that $c < (2q - 1) e / m$ (Assumption 1) implies that $(2q - 1) / c > m / e$. Therefore $F^o < 0$.

Since $V(F)$ increases in $F$ for

$$m - \left( \frac{2q - 1}{2c} \right) \left( \frac{L - (m - e)}{2z} \right) + e < F \leq F^*$$
and decreases in \( F \) for \( F > F^* \), \( F^* \) maximizes the value of the firm in the region of

\[
F > m - \left( \frac{2q - 1}{2c} \right) \left( \frac{L - (m - e)}{2z} + e \right).
\]

In the region of

\[
F < m - \left( \frac{2q - 1}{2c} \right) \left( \frac{L - (m - e)}{2z} + e \right),
\]

\( F^o \) maximizes the value of the firm. This completes the proof. \( \square \)

**Proof of Proposition 4.** At \( F^o \), the informed trading profit is 0, so \( V(F^o) = m \). Then \( V(F^o) > V(F^*) \) if and only if

\[
(C-35) \quad m > m + \frac{1}{2} \left( 1 - \frac{c}{2q - 1} \right) (L - (m - e))
\]

\[
- \left( \frac{z}{2q - 1} \right) \left( 1 - \frac{c}{2q - 1} \right) \left( \frac{m + e - F^*}{2} \right) c,
\]

which is equivalent to

\[
(C-36) \quad m + e - \frac{L - (m - e)}{c} \left( \frac{2q - 1}{z} \right) > F^*.
\]

This completes the proof. \( \square \)

**Proof of Proposition 5.** Plugging \( F^* \) (given by \( (C-27) \)) into \( (C-36) \), \( (C-36) \) can be simplified as

\[
(C-37) \quad \frac{m + e - L - (m - e)}{c} \left( \frac{2q - 1}{z} \right)^2 + \left( \frac{m + e - L - (m - e)}{c} \right) \left( \frac{2q - 1}{z} \right) \left( \frac{2q - 1 - 3c}{2q - 1 - c} \right) (m - e - L)
\]

\[
- L (m - e) \left( \frac{2q - 1 + e}{2q - 1 - e} \right) > 0.
\]

Differentiating the left-hand side of the inequality with respect to \( L \) and \( z \), it is straightforward that it decreases in \( L \) and increases in \( z \). Therefore, the above inequality is more likely to hold when \( L \) is smaller and/or \( z \) is larger. \( \square \)

**Proof of Proposition 6.** From \( (C-34) \), \( F^o = m - ((2q - 1)/c)e \). Clearly, it increases in \( m \) and \( c \) and decreases in \( e \). \( \square \)

**Proof of Proposition 7.** From the proof of Proposition 1, \( F^* \) is given by \( (C-26) \). Define the left-hand side of \( (C-26) \) as

\[
(C-37) \quad y \equiv F^2 + \left( \frac{2q - 1 - 3c}{2q - 1 - c} \right) (m - e) - L (m - e) \left( \frac{2q - 1 + c}{2q - 1 - c} \right).
\]

Since \( L > m - e, F^* > L \) (Proposition 1), and \( 1 - (2c/(2q - 1 - c)) = (2q - 1 - 3c)/(2q - 1 - c) < 1 \),

\[
(C-38) \quad \frac{\partial y}{\partial F^*} = 2F^* + (m - e) \left( \frac{2q - 1 - 3c}{2q - 1 - c} \right) - L > 0.
\]

It is also clear that \( \partial y/\partial L < 0 \). By the implicit function theorem,

\[
\frac{\partial F^*}{\partial L} = -\frac{\partial y/\partial L}{\partial y/\partial F^*} > 0.
\]
In addition,

\[
\frac{\partial y}{\partial (m-e)} = \left(\frac{2q - 1 - 3c}{2q - 1 - c}\right) F - \left(\frac{2q - 1 + c}{2q - 1 - c}\right) L < 0
\]

at \(F^*\) if and only if

\[\text{(C-39)} \quad F^* < \left(\frac{2q - 1 + c}{2q - 1 - 3c}\right) L.\]

Note that

\[y\left(\frac{2q - 1 + c}{2q - 1 - 3c}\right) L > 0.\]

Since \(y\) is convex in \(F\) and

\[y(F^*) = 0, \quad y\left(\frac{2q - 1 + c}{2q - 1 - 3c}\right) L > 0\]

implies (C-39). By the implicit function theorem,

\[
\frac{\partial F^*}{\partial (m-e)} = -\frac{\partial y/\partial (m-e)}{\partial y/\partial F^*} > 0.
\]

Therefore, \(F^*\) increases in \(m\) and decreases in \(e\). \(\square\)

Proof of Proposition 8. Note that \(y\) from (C-37) decreases in \(2c/(2q - 1 - c)\), and that \(2c/(2q - 1 - c)\) increases in \(c\). Then \(c < 2q - 1\) (Assumption 1) implies \(\partial y/\partial c < 0\). From (C-38), \(\partial y/\partial F^* > 0\). Using the implicit function theorem, we have \(\partial F^*/\partial c = -(\partial y/\partial c)/(\partial y/\partial F^*) > 0\). Lastly, \(\partial F^*/\partial q = -(\partial y/\partial q)/(\partial y/\partial F^*)\). It is straightforward that \(\partial y/\partial q > 0\). Then, by (C-38), we have \(\partial F^*/\partial q < 0\). \(\square\)

Proof of Proposition 9. The liquidity premium at \(F^*\) is

\[\pi(F^*) = \left(\frac{z}{2q - 1}\right) \left(1 - \frac{c}{2q - 1}\right) \left(m + e - F^*\right) c\]

(see Proposition 2). Now \(\pi\) clearly decreases in \(F^*\). Since \(F^*\) increases in \(L\) (see Proposition 7), \(\pi\) decreases in \(L\). Note also that

\[
\frac{\partial F^*}{\partial e} = -\frac{\partial y/\partial e}{\partial y/\partial F^*} = -\frac{1 - \frac{2c}{2q-1-c}}{2F^* + (m-e) \left(1 - \frac{2c}{2q-1-c}\right) - L} \left(F^* + L \left(1 + \frac{2c}{2q-1-c}\right)\right).
\]

Then,

\[
\frac{\partial \pi}{\partial e} = \left(\frac{z}{2q - 1}\right) \left(1 - \frac{c}{2q - 1}\right) \left(\frac{c}{2}\right) \left(1 - \frac{\partial F^*}{\partial e}\right) = \left(\frac{z}{2q - 1}\right) \left(1 - \frac{c}{2q - 1}\right) \left(\frac{c}{2}\right) \times \left(\frac{F^* + (m-e) \left(1 - \frac{2c}{2q-1-c}\right) + L \left(\frac{2c}{2q-1-c}\right)}{2F^* + (m-e) \left(1 - \frac{2c}{2q-1-c}\right) - L}\right).
\]

\[
> 0. \square
\]
References


